Integrated Lossy and Lossless Image Coding Based on Lossless Wavelet Transform and Lossy-Lossless Multi-Channel Prediction

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SUMMARY In this report, we propose an integrated lossy and lossless image coding, which is possible to be implemented on one architecture, based on combination of lossless wavelet transform (LWT) and lossy-lossless multi-channel prediction (LLMP). The LWT is applied to divide input signals into frequency sub-bands as octave-band decomposition, whereas the LLMP is designed as a non-separable two-dimensional filter bank including quantization step size and local decoding to enhance coding performance in both lossless coding and lossy coding. Its filter coefficients are determined to minimize total bit rate for lossless coding, and the optimum quantization step size is applied to maximize lossy coding gain. The local decoding is applied to avoid quantization error effect. The experimental results confirm effectiveness of our proposed method.

key words: image, coding, lossless, lossy, filter bank

1. Introduction

Image compression technique [1] is a significant technique to reduce the number of bits required to represent image data. Normally, image compression techniques are classified into two categories: lossy coding and lossless coding. In lossy coding algorithms, high compression ratios are achieved trading off with some distortions. On the other hand, lossless coding algorithms can decode the same image data as an original image without any loss; however, their compression ratios are relatively small. With different advantages, an integrated lossy and lossless coding technique is an attractive coding technique that user can select to encode input data in two different purposes: high quality image or high compression ratio. The combination of them will provide great benefits for many applications such as remote sensing. For example, lossy coding can be employed to scan around the earth since it doesn’t require much bandwidth. If the remote sensing can detect some interesting areas, lossless coding will be operated to provide high quality images.

1.1 Related Work

Up to now, many coding standards introduced from the Joint Photographic Experts Group (JPEG) [2] are well known standards for still image coding. The JPEG based on Discrete Cosine Transform (DCT) [3],[4] is the first standard for still image coding; however this algorithm is limited to only lossy coding. Then the lossless JPEG (L-JPEG) based on Differential Pulse Code Modulation (DPCM) [3] has been proposed for applications that demand a high image quality such as medical imaging, remote sensing, fine art and etc. However, the filter coefficients of DPCM are fixed values, which cannot be adaptive to various characteristics of images. The next standard is called JPEG-LS [5] based on a context-based modeling and a non-linear adaptive predictor referred to Median Edge Detector (MED) in LOCO [6]. The compression ratio of JPEG-LS is better than compression ration of L-JPEG since its filter coefficients are adaptive based on locally varying statistics from image data. So far, many kinds of non-linear adaptive filter have been proposed [7]–[9]. Unfortunately, all of the previous standards are not suitable for progressive resolution transmission because of the absence of the anti-aliasing filter, which can prevent aliasing occurring in low-resolution images.

In opposition, the lossless wavelet transform (LWT) [10]–[17] can provide progressive resolution transmission since low-resolution images are automatically created as a part of its encoding procedure. Various kinds of LWT have been proposed based on lifting structures [18], which are easy and flexible for implementation. Most of the conventional LWT are based on one-dimensional filters so the LWT must be applied to horizontal and vertical dimension independently as a separable two-dimensional filter for image application. Therefore unavoidably remaining correlations still exist [19], so assumption that the resulting transform coefficients are uncorrelated in Shapiro’s paper [22] is incorrect.
1.2 Problem Formulation

Recently, we proposed Lossless Multi-channel Prediction (LMP) [19]–[21] as a non-separable two-dimensional filter bank to utilize remaining correlations of output signals of the LWT since a non-separable two-dimensional filter has a freedom to utilize remaining correlation in horizontal, vertical, and diagonal independently. Its filter characteristics are adaptive to input image’s statistics so we need to attach optimized filter coefficients into the bit streams as side information since its encoder and decoder share the same filter coefficients for each input image signal.

Although the LMP is successful to reduce total bit rate of image data in lossless coding, its lossy coding performance is restricted to only progressive resolution functionality since the LMP is not suitable to add quantization steps directly for providing progressive fidelity functionality. The LMP utilizes remaining correlation by using both extrapolation and interpolation, so its synthesis filter performs as IIR filter, which causes increase of quantization error.

Even though progressive fidelity functionality for LWT has been proposed in many reports [12], [14], [22], [23], they emphasize on quantization part and entropy coding part but they don’t consider transformation part to utilize the still remaining correlation from the LWT.

1.3 Overview of Proposed Method

In this report, we propose an integrated lossy and lossless image coding based on combination of the LWT and lossy-lossless multi-channel prediction (LLMP) as illustrated in Fig. 1(a), and Fig. 1(b) for block diagrams of encoder and decoder, respectively. In encoder part, the LWT is applied to divide input signal \( X_I \) as an octave-band decomposition into 10 hierarchical subbands: \( X_{HL1} \), \( X_{LH1} \), \( X_{HH1} \) for 1st stage, \( X_{HL2} \), \( X_{LH2} \), \( X_{HH2} \) for 2nd stage and \( X_{HL3} \), \( X_{LH3} \), \( X_{HH3} \) for 3rd stage. For example, \( X_{HL2} \) indicates 2nd stage subband that horizontally high passed and vertically low passed.

Note: The LWT can be applied for different no. of stages; however, we apply the LWT for 3 stages in this report.

The LLMP is designed to improve coding performance in both lossless coding and lossy coding. We maximize the lossless coding gain [24] by utilizing the remaining correlations of output signals from the LWT and then the optimum quantization steps are added to maximize lossy coding gain. The local decoding [3] is also applied in analysis filter of LLMP to avoid increase of quantization error, since its synthesis filters perform as IIR filters. According to local decoding, we need to apply LLMP in the following order: LLMP3, LLMP2 and LLMP1, and the decoded signal \( X'_{LL2} \) and \( X'_{LL1} \) are necessary to be decoded to employ in LLMP2 and LLMP1, respectively. In decoder part, we also apply LLMP in the same order as encoder part.

This report is organized as the follows. Section 2 describes basic tools that are used in this report such as signal processing of the LWT to construct analysis filter and synthesis filter, quantization error effect and local decoding. In Sect. 3, we propose the LLMP to improve coding performances in both lossless coding and lossy coding. Optimization methods in both lossless coding and lossy coding are proposed in this section. Experimental results are indicated in Sect. 4 to confirm effectiveness of our proposed method for some images. The first order entropy rate is used to evaluate our proposed method in lossless coding and the coding gain is used to evaluate our proposed method in lossy coding.

2. Basic Tools

2.1 Analysis Filter from Lossless Wavelet Transform (LWT)

The lossless wavelet transforms (LWT), which map integers to integers, have an important influence on research in lossless image compression area since they can guarantee a perfect reconstruction of filter banks and they also provide a progressive resolution transmission functionality. The LWT can be classified into two groups: orthogonal LWT and bi-orthogonal LWT.
Most of orthogonal LWT, such as SP transform [14] and TS transform [15], are based on ST [13], which is the simplest orthogonal LWT, whereas most of bi-orthogonal LWT are based on lifting structures. However, both conventional orthogonal LWT and conventional bi-orthogonal LWT generally perform one dimensional filter bank and they need to apply to horizontal and vertical dimension independently for image application.

Fig. 2(a) illustrates analysis filter for separable two-dimensional bi-orthogonal LWT and Fig. 2(b) illustrates its equivalent expression at 1st stage. The $z_1$ and $z_2$ denote for vertical and horizontal respectively. The $\downarrow 2(n_1)$, $\downarrow 2(n_2)$, $\downarrow 2(n_1, n_2)$ denote down sampling by two in horizontal, in vertical and in both horizontal and vertical, respectively.

By neglecting rounding effect in lifting structure as shown in Fig. 2, the relation between input signals and subband signals in 1st stage is

$$
\begin{bmatrix}
X_{LL1}(z_1, z_2) \\
X_{HL1}(z_1, z_2) \\
X_{LH1}(z_1, z_2) \\
X_{HH1}(z_1, z_2)
\end{bmatrix} = H_1 H_3 H_2 H_1
$$

where

$$
H_1 = \begin{bmatrix}
1/2 & 0 & 1/2 & 0 \\
0 & 1/2 & 0 & 1/2 \\
z_2^{-1/2}/2 & 0 & -z_2^{-1/2}/2 & 0 \\
0 & z_2^{-1/2}/2 & 0 & -z_2^{-1/2}/2
\end{bmatrix}
$$

$$
H_2 = \begin{bmatrix}
1 + P_{01}(z_2) P_{10}(z_2) & 0 \\
0 & 1 + P_{01}(z_2) P_{10}(z_2) \\
z_2^{1/2} P_{01}(z_2) & 0 \\
0 & z_2^{1/2} P_{01}(z_2)
\end{bmatrix}
$$

$$
H_3 = \begin{bmatrix}
1/2 & 1/2 & 0 & 0 \\
z_1^{-1/2}/2 & -z_1^{-1/2}/2 & 0 & 0 \\
0 & 0 & 1/2 & 1/2 \\
0 & 0 & -z_1^{-1/2}/2 & -z_1^{-1/2}/2
\end{bmatrix}
$$

$$
H_4 = \begin{bmatrix}
1 + P_{01}(z_1) P_{10}(z_1) z_1^{-1/2} P_{01}(z_1) & z_1^{-1/2} P_{01}(z_1) \\
0 & 0 \\
0 & 0 \\
0 & 0
\end{bmatrix}
$$

Table 1 Parameters $P_{10}(z)$ and $P_{01}(z)$ for bi-orthogonal LWT.

<table>
<thead>
<tr>
<th>Name</th>
<th>$P_{01}(z)$</th>
<th>$P_{10}(z)$</th>
</tr>
</thead>
<tbody>
<tr>
<td>Ex 1</td>
<td>(-1 -z^2)/2</td>
<td>(z^4 -5z^2 +36z +36 -5z -2)/128</td>
</tr>
<tr>
<td>Ex 2</td>
<td>(-1 -z^2)/2</td>
<td>(z^2 +63z +63 +z^2)/256</td>
</tr>
<tr>
<td>MIT</td>
<td>(z-9-9z^2 +z^3)/16</td>
<td>(z+1)/4</td>
</tr>
<tr>
<td>(9,3)</td>
<td>(-1 -z^2)/2</td>
<td>(-3z^2 +19z +19 -3z^3)/64</td>
</tr>
<tr>
<td>SSKF</td>
<td>(-1 -z^2)/2</td>
<td>(z+1)/4</td>
</tr>
<tr>
<td>CRF</td>
<td>(z-9-9z^2 +z^3)/16</td>
<td>(-z^2 +5z +5 -z^2)/16</td>
</tr>
</tbody>
</table>
Stage. The

Fig. 3

Synthesis filter from separable two-dimensional LWT

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(2D-LWT) at

respectively.

Fig. 3(b) are related to parameters

illustrated in Fig. 3(b). The synthesis FIR filters in

dimensional LWT; whereas its equivalent expression is

Figure 3(a) shows synthesis filter from separable two-

2.2 Synthesis Filter from LWT

Figure 3(a) shows synthesis filter from separable two-

dimensional LWT; whereas its equivalent expression is

in Fig. 3(a) as the following equation:

where

\[
G_1 = \begin{bmatrix}
0 & 1 \\
1 & 0
\end{bmatrix}
\]  

(9)

\[
G_2 = \begin{bmatrix}
1 & -z_2^{-1/2}P_{10}(z_2) \\
-z_2^{-1/2}P_{10}(z_2) & 1 + P_{10}(z_2)P_{01}(z_2)
\end{bmatrix}
\]  

(10)

\[
G_3 = \begin{bmatrix}
-1/2 & 0 \\
1 & 0 \\
0 & -1/2 \\
0 & 1
\end{bmatrix}
\]  

(11)

\[
G_4 = \begin{bmatrix}
1 & -z_1^{-1/2}P_{10}(z_1) \\
-z_1^{-1/2}P_{10}(z_1) & 1 + P_{10}(z_1)P_{01}(z_1)
\end{bmatrix}
\]  

(12)

2.3 Quantization Error Effect and Local Decoding

We demonstrate quantization error effect by simple ex-

ample as illustrated in Fig. 4(a). We can write relation

between input signal \( S_I(z) \) and decoded signal \( S_D(z) \)

in term of quantization noise \( N_q(z) \) as the following

equation:

\[
S_D(z) = S_I(z) + \frac{N_q(z)}{1 - F(z)}
\]  

(13)

where \( F(z) \) indicates predicted filter. The quantization

noise \( N_q(z) \) at decoded image is based on filter \( F(z) \)

since input signal \( S_I(z) \) is not the same as decoded sig-

nal \( S_D(z) \). The local decoding is introduced to avoid

quantization effect as shown in Fig. 4(b). We can write

relation between signal input \( S_I(z) \) and decoded signal \( S_D(z) \) in term of quantization noise \( N_q(z) \) as the

following equation:

\[
S_D(z) = S_I(z) + N_q(z)
\]  

(14)

Fig. 3 Synthesis filter from separable two-dimensional LWT

(2D-LWT) at \( m^{th} \) stage. (b) Its equivalent expression at \( m^{th} \)

stage. The \( \uparrow 2(n_1), \uparrow 2(n_2), \uparrow 2(n_1, n_2) \) denote up sampling by

two in horizontal, in vertical and in both horizontal and vertical, respectively.

Fig. 4 Quantization error effect (a) without local decoding (b) with local decoding.
3. Lossy-Lossless Multi-Channel Prediction (LLMP)

3.1 Analysis Filter of the Lossy-Lossless Multi-Channel Prediction (LLMP)

We propose the LLMP to optimize coding performance in both lossless coding and lossy coding. The structure of the LLMP, as shown in Fig. 5, is mainly divided into two functional parts: lossless optimization part and lossy optimization part.

Lossless optimization algorithm of the LLMP is similar to its of the LMP except local decoding is added to avoid quantization error effects. If lossless mode is selected, all quantization step sizes will be one. Oth-
erwise, optimum quantization step sizes, which depend on synthesis filter of each subband, will be added to maximize lossy coding gain. More details about lossless and lossy optimization are discussed in the next section.

Denoting \{LL, HL, LH, HH\} as \{0, 1, 2, 3\} respectively, we design the LLMP based on a non-separable two-dimension FIR filter \(F_{\text{qm}}\). The \(m\) denotes \(m\)th stage of LLMP which \(m \in \{1, 2, 3\}\). The \(p\) and \(q\) denote subband in the following condition:

\[
p, q = \begin{cases} 
1, 2, 3, & \text{if } m = 2, 1 \\
0, 1, 2, 3, & \text{if } m = 3
\end{cases}
\]  

(15)

We can write relation of subband signals \(x_{pm}(n_1, n_2)\) predicted each other to get predicted error \(e_{pm}(n_1, n_2)\) as the following equation:

\[
e_{pm}(n_1, n_2) = Q_{pm}\left[x_{pm}(n_1, n_2) + R \left[ \sum_{q=0}^{p-1} F_{\text{qm}}[x'_{qm}(n_1, n_2)] \right] \right]
\]

(16)

where \(R[\cdot]\) and \(x'_{pm}(n_1, n_2)\) denote "rounding" function into an integer, and decoded signal of subband "\(p\)" at \(m\)th stage, respectively. \(Q_{pm}[\cdot]\) denotes quantization function of subband "\(p\)" at \(m\)th stage defined as

\[
Q_{pm}[x] = R\left[\frac{x}{\Delta_{pm}}\right]
\]

(17)

where \(\Delta_{pm}\) indicates quantization step size of subband "\(p\)" at \(m\)th stage. The decoded signal \(x'_{pm}(n_1, n_2)\) can be written as the following equation:

\[
x'_{pm}(n_1, n_2) = Q_{pm}^{-1}[e_{pm}(n_1, n_2)]
\]

(18)

where \(Q_{pm}^{-1}[\cdot]\) denotes inverse of quantization function of subband "\(p\)" at \(m\)th stage defined as

\[
Q_{pm}^{-1}[x] = R[x \Delta_{pm}]
\]

(19)

And a non-separable two-dimensional FIR filter \(F_{\text{qm}}\) describes with filter coefficients \(e_{\text{qm}}^{(i,j)}\) as the following equation:

\[
F_{\text{qm}}[x'_{qm}(n_1, n_2)] = \sum_{i} \sum_{j} e_{\text{qm}}^{(i,j)} x'_{qm}(S[n_1-i, N_1], S[n_2-j, N_2])
\]

(20)

where \(N_1\) and \(N_2\) are sizes of subband "\(q\)" at \(m\)th stage in horizontal and vertical, respectively. \(S[\cdot]\) and \(D_{qp}[\cdot]\) denote function for image boundary as the following condition:

We determine coefficients \(e_{\text{qm}}^{(i,j)}\) under the lossless coding gain in the next section.

The combination of the 2D-LWT and the LLMP can be illustrated into an equivalent expression as shown in Fig. 6. The FIR filter of an equivalent expression can be written as the following equation:

\[
\begin{bmatrix}
H'_{L, Lm}(z_1, z_2)
H'_{H, Lm}(z_1, z_2)
H'_{L, Hm}(z_1, z_2)
H'_{H, Hm}(z_1, z_2)
\end{bmatrix} = H_{5m}\cdot H_4H_5H_2H_1
\]

(23)

where \(H_{5m}\) indicates analysis filter of the LLMP at \(m\)th stage that

\[
H_{5k} = \begin{bmatrix}
1 & 0 & -F_{01k}(z_1, z_2) & 1 - F_{11k}(z_1, z_2)
0 & -F_{02k}(z_1, z_2) & 0 & 1 - F_{12k}(z_1, z_2)
0 & 0 & 1 - F_{22k}(z_1, z_2) & 0
1 - F_{23k}(z_1, z_2) & 0 & 1 - F_{33k}(z_1, z_2)
\end{bmatrix}
\]

(24a)

for 1st and 2nd stage of the LLMP \((k = 1, 2)\) and

\[
H_{53} = \begin{bmatrix}
1 - F_{003}(z_1, z_2) & 0 & -F_{013}(z_1, z_2) & 1 - F_{113}(z_1, z_2)
0 & -F_{023}(z_1, z_2) & 0 & 1 - F_{123}(z_1, z_2)
1 - F_{223}(z_1, z_2) & 0 & 1 - F_{333}(z_1, z_2)
0 & 0 & 0 & 1 - F_{233}(z_1, z_2)
\end{bmatrix}
\]

(24b)

for 3rd stage of the LLMP.
3.2 Synthesis Filter of the LLMP

Figures 7(a) and 7(b) show the block diagrams of synthesis filter of the LLMP; whereas its equivalent expression of combination between the LWT and the LLMP illustrated in Fig. 8. The synthesis FIR filters in Fig. 8 are calculated as the following equation.

\[
\begin{bmatrix}
G_{LLm}(z_1, z_2) \\
G_{HLm}(z_1, z_2) \\
G_{LHm}(z_1, z_2) \\
G_{HHm}(z_1, z_2)
\end{bmatrix} = G_{5m} G_4 G_3 G_2 G_1
\]  
(25)

where \( G_{5m} \) indicates synthesis filter of the LLMP at \( m^{th} \) stage that

\[
G_{5m} = H_{5m}^{-1}
\]  
(26)
3.3 Optimization in Lossless Mode

Recently, we defined a new criterion to evaluate system’s performance called lossless coding gain $C_{LSL}$ [24] from ratio between total bit rate from PCM coding $B_{PCM}$ and total bit rate from Lossless coding $B_{LSL}$ as the following equation:

$$C_{LSL} = 20 \log_{10} \frac{B_{PCM}}{B_{LSL}} = C_{LSL}^{*} + c_1$$

(27)

where $C_{LSL}^{*}$ and $c_1$ are defined as the following equation:

$$C_{LSL}^{*} = 10 \log_{10} \frac{\sigma_x^2}{\prod_{b=0}^{B-1} (\sigma_{x_b}^2)^{w_b^{-1}}}$$

(28)

$$c_1 = 20 \log_{10} \frac{\gamma_x}{\prod_{b=0}^{B-1} (\gamma_{x_b}^2)^{w_b^{-1}}}$$

(29)

where $\sigma_x^2$ and $\sigma_{x_b}^2$ are a variance of the signal $x(n_1, n_2)$ and a variance of the subband $x_b(n_1, n_2)$, respectively and $\gamma_x$ and $\gamma_{x_b}$ are constant values fixed by signal’s probability density function from signal $x(n_1, n_2)$ and subband $x_b(n_1, n_2)$, respectively. The filter coefficients in Eq. (20) are considered to maximize lossless coding gain $C_{LSL}$ in Eq. (28) since the parameter $c_1$ in Eq. (29) is constant. We apply the least square auto regression algorithm (LS-AR) [20] to minimize variance in each subband so optimized filter coefficients are solutions from the following equations [24]:

$$\sum_{q=0}^{3} \sum_{i} \sum_{j} c_{qpm}^{(i,j)} \cdot \Phi_m^{(r,k,l-q,i,j)} = -\Phi_m^{(r,k,l-p,0,0)}$$

(30)

where $m \in \{1, 2, 3\}$, $p$ and $r \in \{0, 1, 2, 3\}$, $k$ and $l \in \{0, \pm 1, \pm 2, \cdots \}$ and $\Phi_m^{(r,k,l-q,i,j)}$ is defined as correlation between subband “$q$” and subband “$r$” at $m$th stage of the LLMP as shown in the following equation:

$$\Phi_m^{(r,k,l-q,i,j)} = \sum_{n_2=0}^{N_2-1} \sum_{n_1=0}^{N_1-1} x_{rm}(S[n_1 + k, N_1], S[n_2 + l, N_2])$$

$\cdot x_{qm}(S[n_1 + i, N_1], S[n_2 + j, N_2])$

(31)

### Table 2: Signal processing in the LLMP where $m \in \{1, 2, 3\}$.

<table>
<thead>
<tr>
<th>Signal Processing</th>
<th>$F_{003}$</th>
<th>$F_{00m}$</th>
<th>$F_{01m}$</th>
<th>$F_{02m}$</th>
<th>$F_{11m}$</th>
<th>$F_{12m}$</th>
<th>$F_{22m}$</th>
<th>$F_{31m}$</th>
<th>$F_{32m}$</th>
<th>$F_{33m}$</th>
</tr>
</thead>
<tbody>
<tr>
<td>$C_{003}$</td>
<td>$C_{003}^{(0)} + C_{003}^{(-1)} + C_{003}^{(1)}$</td>
<td>$C_{00m}^{(0)} + C_{00m}^{(-1)} + C_{00m}^{(1)}$</td>
<td>$C_{01m}^{(0)} + C_{01m}^{(-1)} + C_{01m}^{(1)}$</td>
<td>$C_{02m}^{(0)} + C_{02m}^{(-1)} + C_{02m}^{(1)}$</td>
<td>$C_{11m}^{(0)} + C_{11m}^{(-1)} + C_{11m}^{(1)}$</td>
<td>$C_{12m}^{(0)} + C_{12m}^{(-1)} + C_{12m}^{(1)}$</td>
<td>$C_{22m}^{(0)} + C_{22m}^{(-1)} + C_{22m}^{(1)}$</td>
<td>$C_{31m}^{(0)} + C_{31m}^{(-1)} + C_{31m}^{(1)}$</td>
<td>$C_{32m}^{(0)} + C_{32m}^{(-1)} + C_{32m}^{(1)}$</td>
<td>$C_{33m}^{(0)} + C_{33m}^{(-1)} + C_{33m}^{(1)}$</td>
</tr>
</tbody>
</table>

Note: (1) The previous optimization process is not a real-time process since the correlation $\Phi_m^{(r,k,l-q,i,j)}$ in Eqs. (30) and (31) is calculated from statistics of whole image data. The optimized filter coefficients are embedded into bit stream as a side-information. (2) The number of the optimized filter coefficients is based on the chosen order of AR model of the optimization process. The more numbers of the optimized filter coefficients, the better compression result of output signal. In this report, four filter coefficients are selected to perform the LLMP for LL3 subband; whereas, six filter coefficients are selected to perform the LLMP for other subband as shown in Table 2.

3.4 Optimization in Lossy Mode

In lossy mode, we consider the lossy coding gain defined as

$$C_{LSY} = 10 \log_{10} \frac{\sigma_{PCM}^2}{\sigma_{LSY}^2}$$

(32)

where $\sigma_{PCM}^2$ and $\sigma_{LSY}^2$ denote variance of quantization error from PCM coding and from lossy coding, respectively. We can write relation between variance of input signals ($\sigma_x^2$) and variance of their error after PCM ($\sigma_{PCM}^2$) as the following equation [3].

$$\sigma_{PCM}^2 = \varepsilon^2 \cdot 2^{2B_{PCM}} \sigma_x^2$$

(33)

where $\varepsilon$ is a quantization constant depending on statistics of $x$ and on allowed overflow probability and $B_{PCM}$ represents numbers of bits per sample for PCM. For subband coding, we consider to divide input signals into $M$ subbands and $k$th subband is down sampling by $N_k$. We write variance of signals in each subband ($\sigma_{xk}^2$) and variance of their error after quantization in each subband ($\sigma_{ek}^2$) as the following equation.

$$\sigma_{xk}^2 = \varepsilon_k^2 \cdot 2^{2B_{LSYk}} \sigma_{xk}^2$$

(34)

where $\varepsilon_k$ is a quantization constant of $k$th subband and $B_{LSYk}$ represents numbers of bits per sample of $k$th subband. The variance of total quantization error from
subband coding (\(\sigma^2_{SBC}\)) can be written as the following equation [25]:

\[
\sigma^2_{SBC} = \sum_{k=0}^{M-1} \left[ \frac{\varepsilon_k^2 \sigma_k^2 ||G_k||^2}{N_k} \right]
\]

(35)

where

\[
||G_k|| = \sqrt{\sum_{k_1} \sum_{k_2} g^2(k_1, k_2)}
\]

(36)

and \(g_k(k_1, k_2)\) are filter coefficients of the synthesis filter \(G_b\). We can find a relation between \(\sigma^2_{SBC}\) and \(\sigma^2_{e_k}\) as the following equation:

\[
\sigma^2_{SBC} = \sum_{k=0}^{M-1} \left[ \frac{\varepsilon_k 2^{2-BPCM} \sigma^2_{e_k} ||G_k||^2}{N_k} \right]
\]

(37)

Assuming \(\varepsilon_k = \varepsilon\) for all \(k\), we can write a lossy coding gain in Eq. (32) as the following equation:

\[
C_{LSY} = 10 \log_{10} \left( \frac{2^{-2BPCM} \sigma^2_e}{\sum_{k=0}^{M-1} \left[ 2^{-2B_{LSY}} \sigma^2_{e_k} ||G_k||^2 \right]} \right)
\]

(38)

If we apply the quantization step size (\(\Delta_k\)) for \(k\)th subband, we can write relation numbers of bits per sample in lossy coding (\(B_{LSY_k}\)) and in lossless coding (\(B_{LSL_k}\)) by the following equation:

\[
B_{LSY_k} = B_{LSL_k} - \log_2 \Delta_k
\]

(39)

With Eqs. (27), (38), and (39), we can find relation between lossless coding gain in Eq. (27) and lossy coding gain in Eq. (32) as the following equation:

\[
C_{LSY} = C^*_{LSL} - \Omega + c_1
\]

(40)

where \(c_1\) is constant as shown in Eq. (29) and

\[
\Omega = 10 \log_{10} \left( \sum_{k=0}^{M-1} \frac{(\Delta_k^2) ||G_k||^2 N_k^{-1}}{\prod_{k=0}^{M-1} (\Delta_k^2) N_k^{-1}} \right)
\]

(41)

\(C^*_{LSL}\) is already maximized in previous section, so our target is to minimize \(\Omega\). The \(\Omega\) is based on 3 parameters: \(\Delta_b\), \(G_b\), and \(N_b\). The down-sampling \(N_b\) is constant and filter \(G_b\) depends on the LWT, so we calculate the optimum quantization step size (\(\Delta_b\)) to minimize \(\Omega\). We solve the following differential equation

\[
\frac{\partial \Omega}{\partial \Delta_b} = 0; \quad \text{for } \forall \Delta_b
\]

(42)

so we find the optimum quantization steps given by

\[
\frac{\Delta_b}{\Delta_k} = \frac{||G_k||}{||G_b||}
\]

(43)

3.5 Conclusion of Encoding Procedure of the LLMP

To clarify encoding procedures of the LLMP, we summarize them as the following:

1. Calculate filter coefficients for each stage of the LLMP.
2. Select lossless coding or lossy coding.
3. If lossy coding is selected, optimum quantization step sizes are calculated. If lossless mode is selected, all quantization step sizes are selected to be one.
4. Calculate \(e_{LL3}(0,0)\) by using Eq. (16).
5. Calculate \(x_{LL3}(0,0)\) by using Eq. (18).
6. Repeat step 4 and 5 to update \(e_{LL3}\) and \(x_{LL3}\) for all areas in subband LL3.
7. Repeat step 4–6 for subband HL3, LH3, and HH3, respectively.
8. Calculate \(x'_{LL2}\) from subband LL3, HL3, LH3, and HH3 by using filter bank in Eq. (8).
9. Repeat step 4–6 for subband HL2, LH2, and HH2, respectively.
10. Calculate \(x'_{LL1}\) from subband LL2, HL2, LH2, and HH2 by using filter bank in Eq. (8).
11. Repeat step 4–6 for subband HL1, LH1, and HH1, respectively.

3.6 Conclusion of Decoding Procedure of the LLMP

We summarize decoding procedure as the following.

1. Get all filter coefficients of the LLMP and all optimum quantization step sizes.
2. Calculate \(x'_{LL3}\) for all areas by using Eq. (18).
3. Repeat step 2 for subband HL3, LH3, and HH3, respectively.
4. Calculate \(x'_{LL2}\) from subband LL3, HL3, LH3, and HH3 by using filter bank in Eq. (8).
5. Repeat step 2 for subband HL2, LH2, and HH2, respectively.
6. Calculate \(x'_{LL1}\) from subband LL2, HL2, LH2, and HH2 by using filter bank in Eq. (8).
7. Repeat step 2 for subband HL1, LH1, and HH1, respectively.
8. Calculate \(x'_{O}\) from subband LL1, HL1, LH1, and HH1 by using filter bank in Eq. (8).

4. Simulation Results

4.1 Signal Processing in the LLMP for Simulation

In this report, we apply signal processing in the LLMP as shown in Table 2. \(F_{O3m}\) and \(F_{12m}\) are not used in this report since their remaining correlations are very small.

4.2 Simulation Results for Lossless Coding

We test our proposed method by using eight different
kinds of LWT. In this report, we would like to group them into three LWT groups based on their filter characteristic and simulation results. Group 1 consists of S transform and TS since both transforms are orthogonal transforms. Group 2 consists of SSKF, Ex1, Ex2, and (9, 3) which have the same high-pass filter characteristic and groups 3 consists of MIT and CRF, which have the same high-pass filter characteristic too.

We confirm effectiveness of our proposed method with 11 different images by the first order entropy rate defined as the following equation.

\[ H = - \sum_s P_s \log_2 P_s \]  \hspace{1cm} (44)

where \( P_s \) indicates probability of a symbol “s.” For fair comparison, only the subband \( X_{LL3} \) is applied DCPM as the following equation.

\[ y(n_1, n_2) = x(n_1, n_2) - R \left[ \frac{x(n_1 - 1, n_2) - x(n_1, n_2 - 1)}{2} \right] \]  \hspace{1cm} (45)

Table 3 and Table 4 show total bit rate of signal from LWT and signal from LWT + LLMP, respectively.

### Table 3 Total bit rate of signal from LWT for some images.

<table>
<thead>
<tr>
<th>Image name</th>
<th>ST</th>
<th>TS</th>
<th>SSKF</th>
<th>EX1</th>
<th>EX2</th>
<th>(9, 3)</th>
<th>MIT</th>
<th>CRF</th>
</tr>
</thead>
<tbody>
<tr>
<td>Aerial</td>
<td>6.247</td>
<td>5.926</td>
<td>5.79</td>
<td>5.807</td>
<td>5.785</td>
<td>5.814</td>
<td>5.74</td>
<td>5.753</td>
</tr>
<tr>
<td>Girl</td>
<td>5.056</td>
<td>4.72</td>
<td>4.641</td>
<td>4.652</td>
<td>4.64</td>
<td>4.657</td>
<td>4.604</td>
<td>4.611</td>
</tr>
<tr>
<td>Moon</td>
<td>5.196</td>
<td>5.041</td>
<td>5.017</td>
<td>5.023</td>
<td>5.017</td>
<td>5.028</td>
<td>5.016</td>
<td>5.017</td>
</tr>
<tr>
<td>Barbara</td>
<td>5.625</td>
<td>5.181</td>
<td>5.107</td>
<td>5.102</td>
<td>5.107</td>
<td>5.104</td>
<td>4.969</td>
<td>4.957</td>
</tr>
<tr>
<td>Flower</td>
<td>5.718</td>
<td>5.412</td>
<td>5.349</td>
<td>5.327</td>
<td>5.35</td>
<td>5.36</td>
<td>5.259</td>
<td>5.259</td>
</tr>
<tr>
<td>Basketball</td>
<td>6.039</td>
<td>5.847</td>
<td>5.772</td>
<td>5.787</td>
<td>5.769</td>
<td>5.793</td>
<td>5.74</td>
<td>5.754</td>
</tr>
<tr>
<td>Plant</td>
<td>5.76</td>
<td>5.617</td>
<td>5.561</td>
<td>5.57</td>
<td>5.58</td>
<td>5.576</td>
<td>5.531</td>
<td>5.536</td>
</tr>
<tr>
<td>Barbara2</td>
<td>5.451</td>
<td>5.098</td>
<td>5.007</td>
<td>5.01</td>
<td>5.005</td>
<td>5.014</td>
<td>4.955</td>
<td>4.953</td>
</tr>
<tr>
<td>Total</td>
<td>5.633</td>
<td>5.422</td>
<td>5.348</td>
<td>5.356</td>
<td>5.346</td>
<td>5.361</td>
<td>5.313</td>
<td>5.317</td>
</tr>
</tbody>
</table>

### Table 4 Total bit rate of signal from LWT + LLMP for some images.

<table>
<thead>
<tr>
<th>Image name</th>
<th>ST LLMP</th>
<th>TS LLMP</th>
<th>SSKF LLMP</th>
<th>EX1 LLMP</th>
<th>EX2 LLMP</th>
<th>(9, 3) LLMP</th>
<th>MIT LLMP</th>
<th>CRF LLMP</th>
</tr>
</thead>
<tbody>
<tr>
<td>Aerial</td>
<td>5.791</td>
<td>5.733</td>
<td>5.727</td>
<td>5.714</td>
<td>5.734</td>
<td>5.681</td>
<td>5.691</td>
<td></td>
</tr>
<tr>
<td>Moon</td>
<td>4.998</td>
<td>4.993</td>
<td>4.999</td>
<td>5.003</td>
<td>4.999</td>
<td>4.998</td>
<td>4.966</td>
<td></td>
</tr>
<tr>
<td>Barbara</td>
<td>5.064</td>
<td>5.025</td>
<td>5.044</td>
<td>5.054</td>
<td>5.044</td>
<td>4.931</td>
<td>4.905</td>
<td></td>
</tr>
<tr>
<td>Cartoon</td>
<td>6.015</td>
<td>5.989</td>
<td>5.981</td>
<td>5.993</td>
<td>5.979</td>
<td>6.001</td>
<td>5.963</td>
<td></td>
</tr>
<tr>
<td>Flower</td>
<td>5.205</td>
<td>5.16</td>
<td>5.188</td>
<td>5.195</td>
<td>5.183</td>
<td>5.202</td>
<td>5.096</td>
<td></td>
</tr>
<tr>
<td>Basketball</td>
<td>5.707</td>
<td>5.657</td>
<td>5.651</td>
<td>5.663</td>
<td>5.668</td>
<td>5.614</td>
<td>5.627</td>
<td></td>
</tr>
<tr>
<td>Plant</td>
<td>5.561</td>
<td>5.551</td>
<td>5.559</td>
<td>5.543</td>
<td>5.537</td>
<td>5.547</td>
<td>5.533</td>
<td></td>
</tr>
<tr>
<td>Total</td>
<td>5.326</td>
<td>5.296</td>
<td>5.295</td>
<td>5.301</td>
<td>5.294</td>
<td>5.305</td>
<td>5.256</td>
<td></td>
</tr>
<tr>
<td>comparing to table3</td>
<td>0.326</td>
<td>0.126</td>
<td>0.053</td>
<td>0.056</td>
<td>0.052</td>
<td>0.056</td>
<td>0.052</td>
<td>0.052</td>
</tr>
</tbody>
</table>

4.3 Simulation Results for Lossy Coding

We apply lossy coding gain as shown in Eq. (38) to evaluate the coding performance for lossy coding for three different LWT that is representative of each group: ST, SSKF and MIT. We test each LWT with four different conditions:

1. LWT + same quantization step size for each subband.
2. LWT + LLMP + same quantization step size for each subband.
3. LWT + optimum quantization step size for each subband.
4. LWT + LLMP + optimum quantization step size for each subband.

Their simulation results for some images are shown in Tables 5–7. We also calculate the distortion for different bit rate, which is measured by peak signal to noise ratio defined as

\[ PSNR = 10 \log_{10} \left( \frac{255^2}{MSE} \right) \text{ dB} \]  \hspace{1cm} (46)

where MSE denotes the mean square-error between the original image and decoded image. We also plot the relations between PSNR and first entropy rate of ST, SSKF, and MIT for “Barbara” image as illustrated in Figs. 9–11, respectively. Figure 12 shows decoded im-
Table 7  Lossy coding gain for MIT (in dB).

<table>
<thead>
<tr>
<th>Image name</th>
<th>MIT + same Q</th>
<th>MIT + LLMP + same Q</th>
<th>MIT + opt Q</th>
<th>MIT + LLMP + opt Q</th>
</tr>
</thead>
<tbody>
<tr>
<td>Couple</td>
<td>4.5723</td>
<td>5.0029</td>
<td>12.3062</td>
<td>12.8754</td>
</tr>
<tr>
<td>Aerial</td>
<td>0.6995</td>
<td>1.3336</td>
<td>7.7544</td>
<td>8.4704</td>
</tr>
<tr>
<td>Girl</td>
<td>5.8705</td>
<td>6.0657</td>
<td>13.6953</td>
<td>13.8747</td>
</tr>
<tr>
<td>Chest X</td>
<td>1.8738</td>
<td>2.1058</td>
<td>5.4003</td>
<td>5.524</td>
</tr>
<tr>
<td>Moon</td>
<td>3.9946</td>
<td>4.2846</td>
<td>8.8937</td>
<td>9.0572</td>
</tr>
<tr>
<td>Barbara</td>
<td>4.1468</td>
<td>5.174</td>
<td>9.5647</td>
<td>11.084</td>
</tr>
<tr>
<td>Cartoon</td>
<td>1.4868</td>
<td>2.1731</td>
<td>6.5615</td>
<td>7.815</td>
</tr>
<tr>
<td>Flower</td>
<td>4.9727</td>
<td>5.6708</td>
<td>10.8916</td>
<td>12.158</td>
</tr>
<tr>
<td>Basketball</td>
<td>2.4485</td>
<td>3.0717</td>
<td>9.3068</td>
<td>10.452</td>
</tr>
<tr>
<td>Plant</td>
<td>0.8229</td>
<td>1.2566</td>
<td>5.7069</td>
<td>5.8911</td>
</tr>
<tr>
<td>Barbara2</td>
<td>3.6863</td>
<td>4.1886</td>
<td>9.9735</td>
<td>10.4847</td>
</tr>
</tbody>
</table>

Fig. 9  Relation between PSNR and first entropy rate for ST.

Fig. 10  Relation between PSNR and first entropy rate for SSKF.

Fig. 11  Relation between PSNR and first entropy rate for MIT.

Fig. 12  Barbara at different bit rate based on ST + LLMP.

Table 8  Filter coefficients in 3rd stage of LLMP combining with ST for “Barbara.”

<table>
<thead>
<tr>
<th>Signal processing</th>
<th>Filter coefficients</th>
</tr>
</thead>
<tbody>
<tr>
<td>$c_{003}$, $c_{003}$, $c_{003}$</td>
<td>0.1001, -0.536, 0.0737, -0.1624, -0.0717, 0.2195</td>
</tr>
<tr>
<td>$c_{133}$, $c_{133}$, $c_{133}$</td>
<td>0.027, 0.3411, 0.0331, -0.0407, -0.3656, 0.377</td>
</tr>
<tr>
<td>$c_{223}$, $c_{223}$, $c_{223}$, $c_{223}$</td>
<td>-0.1108, 0.1238, 0.0861, -0.1127, 0.1536, 0.2314</td>
</tr>
<tr>
<td>$c_{003}$, $c_{003}$, $c_{003}$, $c_{003}$</td>
<td>-0.5625, -0.6202, 0.2959, -0.0974</td>
</tr>
</tbody>
</table>
We proposed a new integrated lossy and lossless coding based on combination of LWT and LLMP. The LLMP is designed to enhance coding performance in both lossless coding and lossy coding. We confirmed effectiveness of the LLMP with 8 different LWT. The LLMP can reduce total bit rate for all LWT for lossless coding. Moreover, the LLMP can also increase a lossy coding gain, which provides less distortion for lossy coding. However, optimum filter coefficients must be included into bit stream as a side information.

5. Conclusion

We proposed a new integrated lossy and lossless coding based on combination of LWT and LLMP. The LLMP is designed to enhance coding performance in both lossless coding and lossy coding. We confirmed effectiveness of the LLMP with 8 different LWT. The LLMP can reduce total bit rate for all LWT for lossless coding. Moreover, the LLMP can also increase a lossy coding gain, which provides less distortion for lossy coding. However, optimum filter coefficients must be included into bit stream as a side information.

References


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