整数離散コサイン変換の乗算器係数感度の評価

ｻｺﾝ ｳﾄﾞﾑｼﾘ† ﾅﾀﾆ ﾎﾞﾘﾌﾞｰﾝﾋﾗﾝ† ﾏﾘｴﾚｰﾅ ﾊﾟﾗｼｵｽ ﾍﾟﾚｽ† 岩橋政宏†

† 長岡技術科学大学工学部電気系 〒940-2188 新潟県長岡市上富岡町 1603-1

Evaluation of Sensitivity of Integer DCTs

Sakol UDOMSIRI† Natanit BORIBOONHIRAN† Marielena Palacios PEREZ† Masahiro IWAHASHI†

† Faculty of Engineering, Nagaoka University of Technology, Nagaoka-shi, Niigata, 940-2188 Japan

Abstract Nowadays, various types of the “integer” DCT which outputs integer numbers have been widely investigated and its performance have been continuously improved for being able to be used in many areas such as data compression. This report evaluates performance of various integer DCTs and finds the best integer DCT structure from all the possible combinations of various rotation transforms. As a result, it is found that the best integer DCT structure gives the least sensitivity for the finite word length expression of multiplier coefficients and improves PSNR up to 4.29 [dB] for the AR(1) model as an input signal.

Keyword Sensitivity, combination, Integer DCT, evaluation

1. Introduction

Previously, numerous conventional reports have presented various data compression algorithms and one of them is the discrete cosine transform (DCT) which has become the heart of many established international standards such as JPEG and MPEG. The conventional DCT is widely used in many applications but not appropriate for lossless coding because of its real number output. Therefore, this defect persuades many researchers to innovate this modern type of DCT to extend it to the “integer” DCT that outputs integer numbers. It can be used not only for lossy coding but also for lossless coding [1-4]. Normally, the integer DCT is composed of various integer rotation (IR) transforms. The method proposed by Dang et al.[5] can find difference of sensitivity when various rotation transforms are applied for a colored input signal. However, it dealt with only two types of integer DCTs : Fukuma’s integer DCT and Soontorn’s integer DCT in their experiment. At present, there are many kinds of integer DCT [7] but they are not evaluated to find the best one in case of sensitivity.

This report applies the Dang’s technique to find the best integer DCT in respect of multiplier coefficients’ sensitivity by comparing “two” types of integer rotation (IR) transforms illustrated in figure 1. Each of the integer DCTs shown in figure 2 to 7 [7] are evaluated using the AR(1) model as a colored input signal.

2. Integer Rotation Transform

Generally, integer approximation of the ordinal DCT preserves all the basic mathematical properties of the original real-valued transform matrices. In this report, we refer to the Given-Jacobi rotation matrices as follows,

$$H = \begin{pmatrix} \cos\theta & -\sin\theta \\ \sin\theta & \cos\theta \end{pmatrix}, \qquad (1)$$

$$X = \begin{pmatrix} x(0) & x(1) \end{pmatrix}^T, \quad Y = \begin{pmatrix} y(0) & y(1) \end{pmatrix}^T$$

where X and Y are an input vector and rotated output vector, respectively. From equation (1), it can be factorized to be the lifting matrices shown as follows.

Type A

$$H = \begin{pmatrix} 1 & 0 \\ c & 1 \end{pmatrix}\begin{pmatrix} 1 & b \\ 0 & 1 \end{pmatrix}\begin{pmatrix} 1 & 0 \\ a & 1 \end{pmatrix} \qquad (2)$$

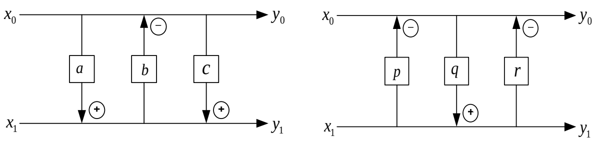
Type B

$$H = \begin{pmatrix} 1 & p \\ 0 & 1 \end{pmatrix}\begin{pmatrix} 1 & 0 \\ q & 1 \end{pmatrix}\begin{pmatrix} 1 & r \\ 0 & 1 \end{pmatrix} \qquad (3)$$

$$a = c = (1-\cos\theta)\sin^{-1}\theta, \quad b = -\sin\theta$$
$$p = r = -(1-\cos\theta)\sin^{-1}\theta, \quad q = \sin\theta$$

The “a”, “b” and “c” are multiplier coefficients of the IR type A and the “p”, “q” and “r” are multiplier coefficients of the IR type B illustrated in figure 1. All the integer

DCT types include different number of IRs and each IR includes three multiplier coefficients. Normally, for lossy coding, each multiplier coefficient is real number identical to the conventional DCT. On the contrary, for lossless coding, the word length of each multiplier coefficient is truncated to be finite word length. However, some approximation error may occur from the finite word length expression of each multiplier coefficient. Therefore, in this report, we focus on evaluation and comparison of sensitivity of each integer DCT and apply the Dang et al.[5]'s technique to find the best integer DCT in aspect of average sensitivity and PSNR.



(a) Type A (b) Type B

Fig. 1 Two types of integer rotation (IR) transforms.

3. Integer DCT Algorithms

This report deals with recently reported six types of integer DCTs shown in figure 2 to 7 for comparison. Those figures show the signal flow graph for the forward transform of each integer DCT. A dotted line denotes negative sign. Each of the integer DCT contains the integer rotation (IR) transform shown in figure 1. The permutation (P) does not contain any multiplier coefficients but just only rearrange input or output of transform algorithm.

In fact, different structure of each integer DCT tells us about difference of its performance such as hardware complexity, power consumption and so on. The more the number of multiplier coefficients, the more complexity and power consumption. Table 1 illustrates the number of IRs and multiplier coefficients of each integer DCT. As the Bin DCT-IIL has the least number of multiplier coefficients, it is advantageous to hardware complexity. Meanwhile, the QR-based LDCT-II has the most complex structure because of the most number of multipliers.

Table 1 The number of IRs and multipliers of the DCTs.

Type	The number of IRs	The number of multipliers
BinDCT-IIL	3	9
BinDCT-IIC	5	15
BinDCT-IIS	5	15
BinDCT-IV	7	21
LDCT-II	8	24
IntDCT-II	5	15

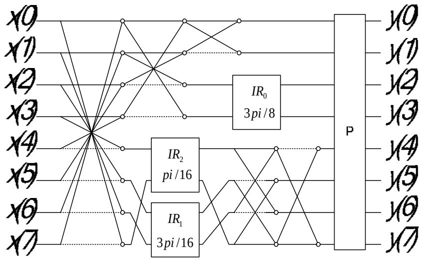


Fig. 2 BinDCT-IIL algorithm

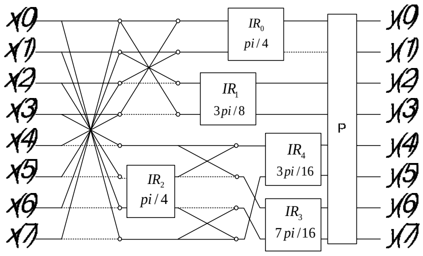


Fig.3 BinDCT-IIC algorithm

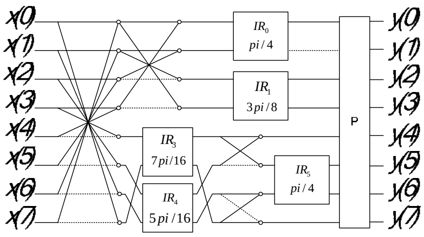


Fig.4 BinDCT-IIS algorithm

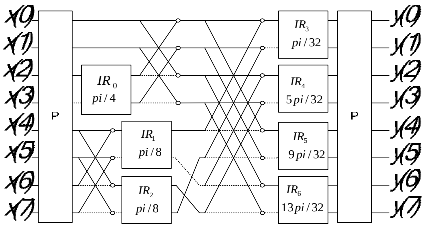


Fig. 5 BinDCT-IV algorithm

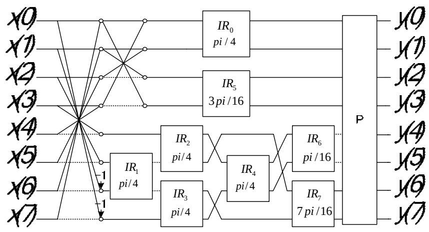


Fig.6 QR-based LDCT-II algorithm

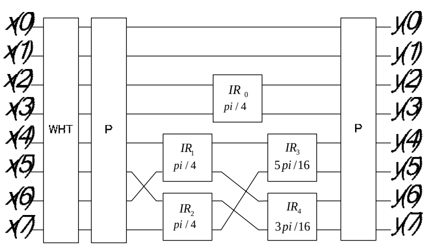


Fig.7 IntDCT-II algorithm

4. Sensitivity

While a multiplier coefficient h in either of the forward transform H or the backward transform H^{-1} is truncated into $\hat{h}_k$, k={0,1,⋯.K-1}, an approximation error between the input signal X and the reconstructed signal $\hat{X}$ is observed by the Root Mean Square Error (RMSE) given by Eq.(4) and Peak Signal to Noise Ratio (PSNR) given by Eq. (5) where 2^R-1 is the maximum gray level number and $N_1 N_2$ denotes size of an input matrix.

$$RMSE = \sqrt{\frac{1}{N_1 N_2}\sum_{n_1=0}^{N_1-1}\sum_{n_2=0}^{N_2-1}\left(X(n_1,n_2)-\hat{X}(n_1,n_2)\right)^2} \qquad (4)$$

$$PSNR = 20\log_{10}\frac{2^R-1}{RMSE} \quad dB \qquad (5)$$

Normally, The PSNR degradation depends on word length of each multiplier coefficient. The longer bit length for representation, the better is the performance. However, this may be at the cost of increased computational complexity.

The sensitivity of each multiplier coefficient to finite word length expression is experimentally evaluated from the process shown in figure 8 and expressed by the equation as follows.

$$S_k = \frac{\Delta\sigma_k}{\Delta h_k}, \quad k = 0,1,\ldots,K-1 \qquad (6)$$

The average sensitivity of each integer DCT combination is defined by

$$\bar{S} = \prod_{k=0}^{K-1}\sqrt[K]{S_k} \qquad (7)$$

$$\Delta h_k = \hat{h}_k - h_k, \qquad h_k \in \{a,b,c,p,q,r\}$$

where K is the number of multiplier coefficients and $\Delta\sigma_k$ denotes standard deviation of output error.

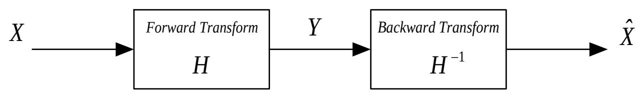


Fig.8 Evaluation of the sensitivity

5. Experimental Results

The AR(1) model with $\rho = 0.9$ is used as a colored input signal for this experiment. Actually, existing integer DCT includes only type A of IR. Therefore, in this experiment, Dang's technique [5] is applied to find the best combination of the integer DCTs by using various lifting structures of IRs. This report investigates two types of lifting structures illustrated in figure 1 and shows the difference of average sensitivity and PSNR of each combination.

The PSNR versus the combination number "C" of each integer DCT algorithm is shown in figure 9 to 14. Denoting N_{RT} as the number of IRs in an integer DCT, the combination number C is defined by

$$C = \sum_{n=1}^{N_{RT}} c_n 2^{n-1}, \qquad c_n = \begin{cases} 0 & (IR_n \quad is \quad type\ A) \\ 1 & (IR_n \quad is \quad type\ B) \end{cases} \qquad (8)$$

Table 3 to 8 show the average sensitivity of the best combination (proposed method), the existing method (c_n=0 for all n) and the worst case. As a result, we can find the best combination of each of the integer DCTs in respect of the average sensitivity and PSNR.

Our experiment focuses on comparison of the average sensitivity and PSNR and also determination of the best combination of each integer DCT. We can see that the average sensitivity is inversely proportion to PSNR. As a result, from table 3 to 8, comparison of PSNR for the best combination of each integer DCT is found that the IntDCT-II algorithm is the best combination because it has the least average sensitivity and the most PSNR. However, if all of the integer DCTs are evaluated in case of PSNR improvement, it is obviously found that the best combination ($N = 28$) of BinDCT-IV algorithm improves PSNR by 4.29 dB superior to the existing combination ($N = 1$) and it is the best PSNR improvement.

Table 2 The number of combinations of integer DCTs.

Type	The number of combinations
BinDCT-IIL	8
BinDCT-IIC	32
BinDCT-IIS	32
BinDCT-IV	128
LDCT-II	256
IntDCT-II	32

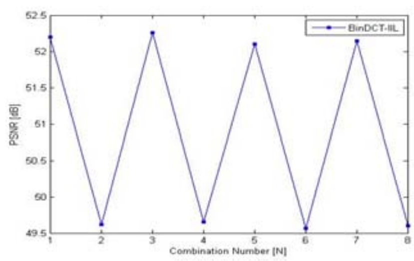


Fig.9 PSNR versus 8 combinations of BinDCT-IIL.

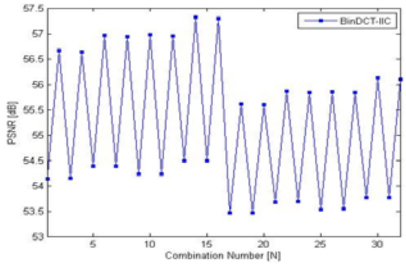


Fig.10 PSNR versus 32 combinations of BinDCT-IIC.

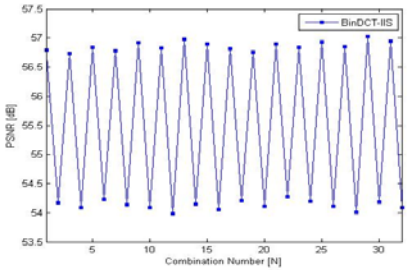


Fig.11 PSNR versus 32 combinations of BinDCT-IIS.

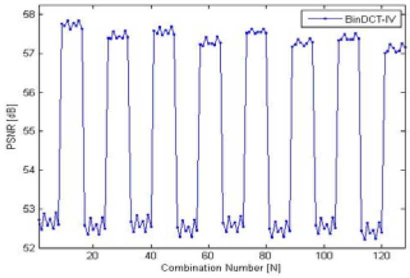


Fig.12 PSNR versus 128 combinations of BinDCT-IV.

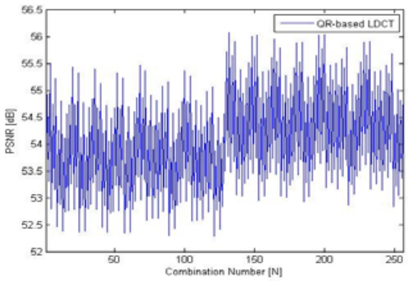


Fig.13 PSNR versus 256 combinations of QR-based LDCT-II.

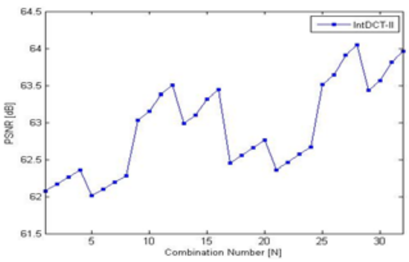


Fig.14 PSNR versus 32 combinations of IntDCT-II.

Table 3 Results of BinDCT-IIL

N	IR2	IR1	IR0	Average sensitivity	PSNR	Remark
1	A	A	A	2.34	52.67	Best
1	A	A	A	2.34	52.67	Existing
8	B	B	B	2.53	50.31	Worst

Table 4 Results of BinDCT-IIC

N	IR4	IR3	IR2	IR1	IR0	Average sensitivity	PSNR	Remark
6	A	A	B	A	B	1.69	60.76	Best
1	A	A	A	A	A	2	58.6	Existing
27	B	B	B	B	B	1.88	58.18	Worst

Table 5 Results of BinDCT-IIS

N	IR4	IR3	IR2	IR1	IR0	Average sensitivity	PSNR	Remark
23	B	A	B	B	B	1.59	57.05	Best
1	A	A	A	A	A	1.64	56.95	Existing
10	A	B	A	B	A	1.78	54.59	Worst

Table 6 Results of BinDCT-IV

N	IR6	IR5	IR4	IR3	IR2	IR1	IR0	Average sensitivity	PSNR	Remark
28	A	A	B	B	A	B	B	1.79	57.81	Best
1	A	A	A	A	A	A	A	2.11	53.52	Existing
37	A	B	A	A	B	A	B	2.14	53.47	Worst

Table 7 Results of QR-based LDCT-II

N	IR7	IR6	IR5	IR4	IR3	IR2	IR1	IR0	Average sensitivity	PSNR	Remark
132	B	A	A	A	A	A	B	B	1.58	56.05	Best
1	A	A	A	A	A	A	A	A	1.72	53.05	Existing
125	B	B	B	B	B	B	A	A	1.83	52.52	Worst

Table 8 Results of IntDCT-II

N	IR4	IR3	IR2	IR1	IR0	Average sensitivity	PSNR	Remark
32	B	B	B	B	B	1.31	63.83	Best
1	A	A	A	A	A	1.16	62.08	Existing
1	A	A	A	A	A	1.16	62.08	Worst

6. Conclusions

This report experimentally evaluated and compared six types of the integer DCT algorithms in aspect of sensitivity and PSNR. As a result, it is found that the best combination of IntDCT-II algorithm has the least average sensitivity and the most PSNR compare to other experimented integer DCTs. The best PSNR improvement is the best combination of BinDCT-IV algorithm in which it improves the PSNR up to 4.29 dB by comparing with the existing combination and using AR(1) model as the input signal.

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