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DRAFT

LOSSY COMPRESSION OF SPARSE HISTOGRAM IMAGE

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ABSTRACT

In this paper, a lossy data compression for a sparse histogram image signal is proposed. It is extended from an existing lossless coding which is based on a lossless histogram packing and a lossless coding. We introduce a lossy mapping, which has less computational load than the rate-distortion optimized Lloyd-Max quantization, and combine it with a lossless coding. It was confirmed that the proposed method attains higher performance in the rate-distortion plane than existing methods. This is because it can utilize histogram sparseness of images, and also its inverse mapping does not magnify quantization noise.

Index Terms— coding, lossless, histogram, image

1. INTRODUCTION

Recently, higher flexibility of image representations are required in advanced video technologies. The high dynamic range (HDR) pixel value representation is considered to be one of the most attractive approaches in this area. It has been crucial to compress its data volume to store and transmit since it consumes huge space of memory [1,2].

A pixel value in HDR has longer bit depth than a current standard (i.e. 8 bit per pixel which is equivalent to 2^8 available tones for a pixel). In this case, the sparseness in histogram has been becoming a new point of view to be considered [3-5]. Due to huge variety of available tone slots, not all the bins in a histogram are utilized in general. It makes a histogram 'sparse'.

To make the most of this unique property of sparse histogram images, the histogram packing has been introduced [3]. It maps a set of original values into another set so that the sparse histogram becomes dense. It contributes to reduce data volume in combination with a lossless coding based a transform or a prediction [4,5]. However, it has been limited to 'lossless', so far.

In this paper, we propose a 'lossy' data compression for sparse histogram images. A direct expansion is a combination of the lossless histogram packing and a 'lossy' coding such as JPEG 2000 and JPEG LS. When a lossy JPEG 2000 is utilized as the lossy coding, noise due to quantization of transformed coefficients is magnified by the inverse procedure of the histogram packing in a decoder. As a result, high quality of reconstructed images can't be obtained.

On the contrary, when a lossy JPEG-LS in near lossless mode is utilized, its rate control is limited to coarse and high PSNR at high bit rate can't be realized. This is because the prediction based near lossless coding has a constrain that the quantization step size must be an integer.

In this paper, to attain both of high image quality and fine rate control, we introduce a 'lossy' mapping and combine it with a 'lossless' coding. In our method, any kind of coding algorithms such as JPEG 2000 based on an integer transform, and JPEG LS based on a prediction can be used as a lossy coding.

As a lossy mapping, the local packing of histogram introduced in [6] can be a candidate as a rate optimized method. However it is not always optimum in the rate-distortion sense. The Lloyd-Max quantization in [7] can be another candidate. However both of them are not adaptive to the histogram sparseness, and also they require heavy computational load to find the optimum solution. Unlike these methods, a lossy mapping used in this paper is quite simple to implement without degrading rate distortion performance of the Lloyd-Max quantization.

In our experiments, we confirm that the proposed method attains higher performance in the rate-distortion plane than the existing methods. This is because it can utilize histogram sparseness of images, and also its inverse mapping does not magnify quantization noise.

2. EXISTING METHOD

A sparse histogram image discussed in this paper is defined. Existing methods and their problems are described.

2.1. Sparse Histogram Image

Fig.1(a) illustrates a histogram of a standard image 'Lena'. Intensity of a pixel value $x(n_1,n_2)$ at vertical location n_1 and horizontal location n_2 is represented with an 8 bit integer. This image does not use all the available 2^8 slots. Defining the histogram sparseness by

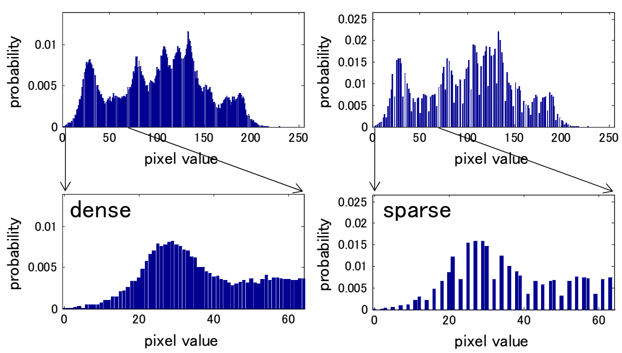
$$\alpha = \frac{|\{x \mid \Pr(x) \neq 0\}|}{\max\{x\} - \min\{x\} + 1} \cdot 100 \tag{1}$$

it has α=95.2 [%] sparseness. In the equation, |X| denotes the cardinal number of a set X. In this case, it means the number of non-zero histogram bins. When we apply

$$x'(n_1, n_2) = HstEq\left[Floor\left[x(n_1, n_2)/\beta\right]\right] \cdot \beta \tag{2}$$

to this image, its sparseness becomes α=53.5 [%] for β=1.8. In the equation, '*Floor*[*p*]' means the largest integer not greater than *p*, and '*HstEq*' means histogram equalization (We used 'histeq' function in MATLAB). This is an example of the 'sparse histogram image'.

In HDR representation, an image tends to be sparse in this sense due to its huge number of histogram bins. It often occurs by an image pre-processing such as a histogram modification, a tone mapping, a Gamma correction, an extraction of a region of interest, and so on. In this paper, we consider the histogram sparse image like this.



(a) α=95.2, β=1 (b) α=53.5, β=1.8

Fig.1 Histogram of an image 'Lena'.

2.2. Lossless Mapping and Lossless Coding

The histogram packing introduced in [3] converts the sparse histogram into dense one. It maps a sparse set of original values *x* into a dense set of values *y* as

$$y(\mathbf{n}) = M \circ x(\mathbf{n}), \quad \mathbf{n} = [n_1, n_2] \tag{3}$$

where $M \circ$ denotes a mapping as an operation which holds

$$\begin{cases} \hat{x}(\mathbf{n}) - x(\mathbf{n}) = 0 \\ \hat{x}(\mathbf{n}) = M^{-1} \circ M \circ x(\mathbf{n}) . \end{cases} \tag{4}$$

Even though this lossless mapping does not reduce the 1st order entropy itself, it does reduce data volume of sparse images when it is combined with a lossless coding as illustrated in Fig.2 [4,5]. However it has been limited to 'lossless', and therefore the bit rate can't be controlled.

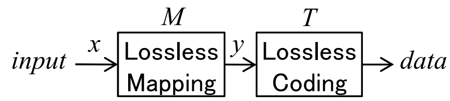
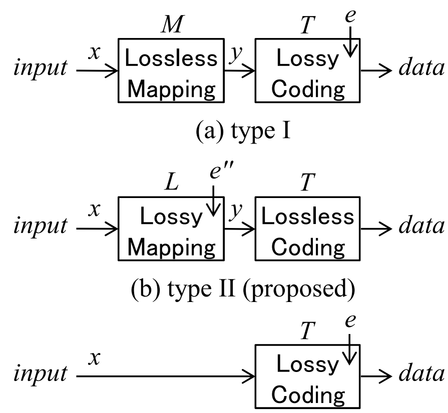


Fig.2 'Lossless' encoding for sparse histogram images.



(a) type I

(b) type II (proposed)

(c) type III (existing)

Fig.3 'Lossy' encoding for sparse histogram images.

2.3. Lossy Mapping and Lossless Coding

Our purpose is to construct a 'lossy' coding for histogram sparse images. A direct expansion is a simple combination of a lossless mapping and a lossy coding as illustrated in Fig.3(a). When a transform is used in the lossy coding like JPEG 2000, quantization noise generated inside the lossy coding is magnified by the inverse mapping. It degrades reconstructed image quality.

This case is detailed as below. A mapped signal $y(\mathbf{n})$ is transformed by $T \circ$, and quantized with a step size $q_{\mathbf{m}}$ as

$$\begin{aligned} w(\mathbf{m}) &= Round\left[T \circ y(\mathbf{n}) \cdot q_{\mathbf{m}}^{-1}\right] \\ &= T \circ y(\mathbf{n}) \cdot q_{\mathbf{m}}^{-1} + e(\mathbf{m}) \end{aligned} . \tag{5}$$

where '*Round* [*p*]' means rounding to the integer nearest to *p*. It is reconstructed by

$$\begin{aligned} \hat{y}(\mathbf{n}) &= T^{-1} \circ \left(w(\mathbf{m}) \cdot q_{\mathbf{m}}\right) \\ &= T^{-1} \circ \left(T \circ y(\mathbf{n}) + e(\mathbf{m}) \cdot q_{\mathbf{m}}\right) \end{aligned} \tag{6}$$

where $e(\mathbf{m})$ denotes the quantization noise in transform domain. As a result, we have the reconstructed signal as

$$\begin{aligned} \hat{x}(\mathbf{n}) &= M^{-1} \circ \left(T^{-1} \circ \left(T \circ M \circ x(\mathbf{n}) + e(\mathbf{m}) \cdot q_{\mathbf{m}}\right)\right) \\ &= x(\mathbf{n}) + M^{-1} \circ T^{-1} \circ e(\mathbf{m}) \cdot q_{\mathbf{m}} . \end{aligned} \tag{7}$$

This means that the probability density function of the quantization noise $e(\mathbf{m})$ in transform domain is scattered by the inverse transform $T^{-1} \circ$.

Similarly, when a near lossless prediction is used like JPEG LS, we have a reconstructed image as

$$\hat{x}(\mathbf{n}) = x(\mathbf{n}) + M^{-1} \circ e'(\mathbf{n}) \cdot q_{\mathbf{m}} \tag{8}$$

where $e'(\mathbf{n})$ denotes the quantization noise in spatial domain. In both cases, the error is magnified by the inverse mapping $M^{-1} \circ$. Therefore high quality reconstructed images can't be obtained.

3. PROPOSED METHOD

A new lossy coding for sparse histogram image is described. It can utilize the histogram sparseness of images, and its mapping does not magnify the quantization noise.

3.1. Lossy Mapping and Lossless Coding

Fig.3(b) illustrates our coding scheme. We utilize a lossy mapping, and combine it with a lossless coding. In this case, we have the reconstructed signal as

$$\begin{aligned} \hat{x}(\mathbf{n}) &= L^{-1} \circ T^{-1} \circ \left(T \circ L \circ x(\mathbf{n})\right) \\ &= x(\mathbf{n}) + e''(\mathbf{n}) \end{aligned} \tag{9}$$

where

$$\begin{cases} T^{-1} \circ T \circ y(\mathbf{n}) = y(\mathbf{n}) \\ L^{-1} \circ L \circ x(\mathbf{n}) = x(\mathbf{n}) + e''(\mathbf{n}) \end{cases} \tag{10}$$

and $L \circ$ denotes a lossy mapping. Note that the noise $e''(\mathbf{n})$ is generated in spatial domain by the mapping, and therefore the noise is not magnified by the inverse mapping.

3.2. Weighted Median Cut Quantization (WMCQ)

Procedure of the lossy mapping is detailed as below. Due to similarity to the median cut quantization, we refer to it as 'weighted median cut quantization (WMCQ)'. It reduces 2^N kinds of tones for a pixel value x to L tones ($2^N > L$), and it can utilize 'sparseness' of the histogram $H(x)$. Forward mapping and backward mapping are performed with tables Q and R as

$$\begin{cases} y(\mathbf{n}) = \quad L \circ x(\mathbf{n}) = Q\left(x(\mathbf{n})\right) \\ \hat{x}(\mathbf{n}) = L^{-1} \circ y(\mathbf{n}) = R\left(y(\mathbf{n})\right) \end{cases} \tag{11}$$

and therefore these tables should be prepared beforehand.

Step 1. Calculate a histogram $H(x)$ of integer pixel values $x \in [0, 2^N)$. Note that not all the 2^N bins but only N_e bins have non-zero values for a sparse image where $N_e < 2^N$.

Step 2. To reduce the number of bins from 2^N to L, if neighboring s non-zero bins are unify into one class for

$$s = \lfloor N_e / L \rfloor, \tag{12}$$

there are still remaining $N_e - s \cdot L = N_h$ non-zero bins. Therefore, unify s+1 bins into one for $N_h(s+1)$ non-zero bins of $H(x)$, and unify s bins into one for $(L-N_h)s$ non-zero bins. This simple procedure fully utilizes the histogram sparseness of input images.

Step 3. Calculate the tables Q and R in (11) as below. It unifies s+1 bins for the first N_h classes, and unifies s bins for the rest L-N_h classes as an example.

$$Q(m_i) = \begin{cases} \left\lfloor \dfrac{i}{s+1} \right\rfloor & , \quad i \in \left[\, 0, N_h(s+1) \,\right) \\ \left\lfloor \dfrac{i - N_h(s+1)}{s} \right\rfloor + N_h, & i \in \left[N_h(s+1), N_e\right) \end{cases} \tag{13}$$

and

$$R(n) = \begin{cases} \dfrac{\sum_{i=(s+1)n}^{(s+1)n+s} H(m_i) \cdot m_i}{\sum_{i=(s+1)n}^{(s+1)n+s} H(m_i)} & , \quad n \in \left[0, N_h\right) \\ \dfrac{\sum_{i=sn+N_h}^{sn+N_h+s-1} H(m_i) \cdot m_i}{\sum_{i=sn+N_h}^{sn+N_h+s-1} H(m_i)} & , \quad n \in \left[N_h, L\right) \end{cases} \tag{14}$$

where

$$m_i = \left\{ x \in [0, 2^N) \;\middle|\; H(x) \neq 0 \right\}, \; i \in [0, N_e) \tag{15}$$

indicates location of non-zero bins in the original histogram.

The table Q is used to quantize 2^N original pixel tones into L tones considering histogram sparseness. The right-hand side of (15) calculates center of mass of each bin in the histogram. It is rounded and used in the decoder.

4. EXPERIMENTS

Computational load of WMCQ is examined, and the three methods in Fig.3 are compared.

4.1. Computational Load

Table I summarizes computational time for quantization from 2^{12} tones to 2^B tones. It compares the Lloyd-Max quantization in [8] and the WMCQ described in **3.2**. It was evaluated on a 2.8 GHz Core Duo computer with MATLAB. As a result, the 'WMCQ' was observed to be faster than the 'Lloyd-Max'.

Table I Computational time for quantization [m sec].

bit depth B	1	2	3	4	5	6	7	8	9	10	11
Lloyd-Max	28	72	73	134	218	230	159	25	18	23	23
WMCQ	15	13	13	13	13	13	13	15	14	14	17

4.2. Performance in Rate-Distortion Plane

Fig.4(a) compares the three methods in Fig.3 for a 16 bit depth (2^{16} available tones) image 'Cafe'. It was indicated that 'Proposed ^{2K}' with lossless JPEG 2000 (2K) and WMCQ in Fig.3(b) is the best of all, followed by 'JPEG 2K' with the 5/3 wavelet and the bit truncation in Fig.3(c). 'Packing+JPEG 2K' with the histogram packing [3] and the lossy JPEG 2K in Fig.3(a) was observed to be the worst.

In Fig.4(b), 'JPEG LS' and 'Proposed ^{LS}' was observed to be almost the same. However, the rate control of 'Proposed ^{LS}' is fine. It makes it possible to smoothly control at bit rates higher than 9.5 [bpp].

Fig.5 summarizes the results for a 12 bit depth medical 'CT image'. For this 'sparse' image (α=45.0%), superiority of the 'Proposed' at lossless point (PSNR=∞) was confirmed. It utilizes the histogram sparseness for data compression. Furthermore, unlike the 'JPEG LS', the 'Proposed ^{LS}' can control bit rates higher than 5.9 [bpp].

5. CONCLUSIONS

We extended the histogram packing from lossless to lossy, and combined it with a lossless coding. In our experiments, JPEG 2000 or JPEG LS international standard was used as the lossless coding, and the weighted median cut quantization was introduced as the lossy mapping. It was confirmed that the proposed method is the best in PSNR for JPEG 2000 case, and the proposed method realizes smooth rate control at high bit rates for both of JPEG 2000 and JPEG LS cases.

6. REFERENCES

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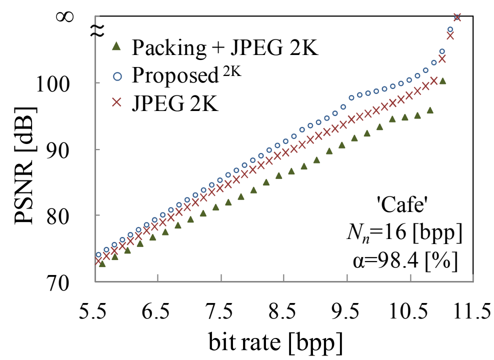
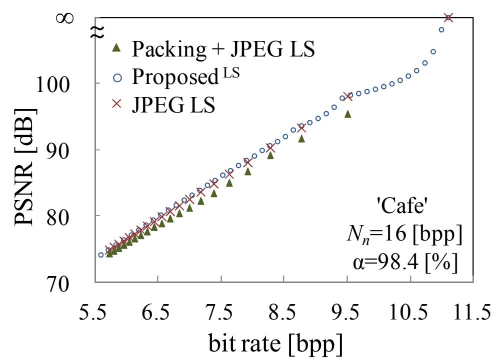
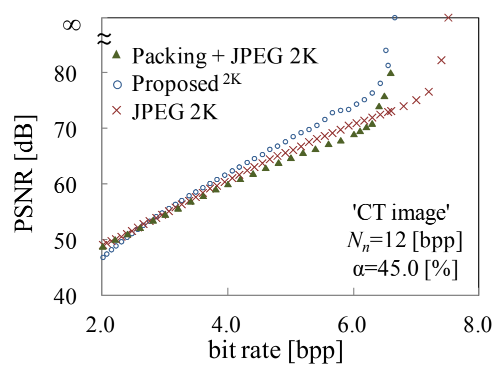


(a) JPEG 2000

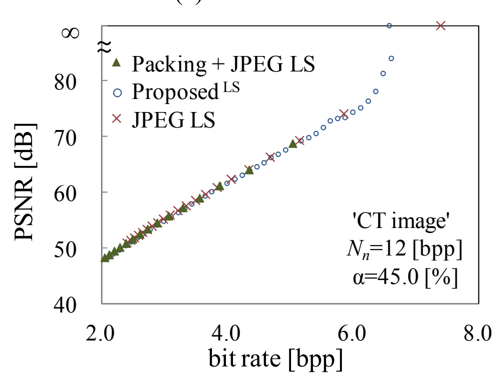


(b) JPEG LS

Fig.4 Rate distortion curves of a dense image.



(a) JPEG 2000



(b) JPEG LS

Fig.5 Rate distortion curves of a sparse image.