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Fine Rate Control and High SNR Coding for Sparse Histogram Images

Masahiro Iwahashi
Dept. of Electrical Engineering
Nagaoka University of Technology
Nagaoka, Niigata, Japan

Hiroyuki Kobayashi
Tokyo Metropolitan College of Industrial Technology
Shinagawa, Tokyo, Japan

Hitoshi Kiya
Dept. of System Design
Tokyo Metropolitan University
Hino, Tokyo, Japan

***Abstract*— In this report, we propose a lossy compression algorithm which can utilize the 'histogram sparseness' of pixel values of an input image signal. The bit depth of pixel values has becoming longer than conventional ones in high dynamic range images. Their histogram of pixel values tends to be 'sparse', - not all the histogram bins are used. We extend an existing lossless coding based on the histogram packing to a lossy coding, introducing a non-uniform quantization especially for histogram sparse images. It was confirmed that the proposed method attains finer control of the bit rate and also higher SNR in the rate-distortion curve comparing to the existing method.**

Keywords- coding, lossless, histogram, image

I. INTRODUCTION

High dynamic range (HDR) pixel value representation has becoming one of the most attractive approaches in advanced video technologies with higher flexibility of image representations. Since it consumes huge memory space in its original data form, it has been crucial to compress its data volume for storage and transmission [1,2].

In an HDR image signal, the bit depth of pixel values is longer than that of a current 8 bit in which a pixel value is represented as one of 2^8 =256 available tones. Due to large variety of available tone slots, not all the histogram bins are utilized in HDR. It makes its histogram 'sparse', and therefore the histogram sparseness is becoming a new point of view to be considered in designing an image encoder [3-5].

The histogram packing has been introduced to make the most of this unique property [3]. It maps an original value set to another one so that the sparse histogram becomes dense. It was reported to contribute to data volume compression in combination with a lossless coding based a transform or a prediction [4,5]. However, it has been limited to 'lossless'.

In this report, we propose a 'lossy' data compression for sparse histogram images. A simple combination of the histogram packing and a 'lossy' encoder such as JPEG-2000 (2K) [9] or JPEG-LS (LS) [10] can be one of solutions to our purpose. However, when the lossy 2K is utilized as the lossy encoder, quantization noise generated in an encoder is magnified by the inverse procedure of the histogram packing in a decoder. As a result, 'high SNR' (signal to noise ratio) of the reconstructed image can't be attained. On the contrary, when the lossy LS (near lossless mode) is applied, there is a difficulty in 'fine rate control' of the bit rate (or data file size).

We introduce a non-uniform quantization (NUQ) combining with a 'lossless' encoder to attain both of 'high SNR' and 'fine rate control', especially for sparse histogram images. In our method, implementation of NUQ is relatively simple, and any kind of coding algorithms such as lossless JPEG 2K or LS can be applied as a lossless encoder.

As the NUQ, we can use the local packing of histogram introduced in [6]. However it is not always optimum in the rate-distortion sense. The Lloyd-Max quantization in [7] can be an alternative choice. However both of them are not adaptive to the histogram sparseness, and also they require heavy computational load to find the optimum solution.

Unlike these methods, the NUQ introduced in this report is quite simple to implement without degrading rate distortion performance of the Lloyd-Max quantization. In our experiments, we confirm that the proposed method attains both of 'high' SNR and 'fine' rate control in the rate-distortion curve comparing to the existing method. This is because it utilizes histogram sparseness of images, and also its decoding process does not magnify errors due to the quantization.

II. EXISTING METHOD

A sparse histogram image discussed in this report is defined. Existing methods and their problems are described.

A. Sparse Histogram Images

First of all, we define the 'histogram sparseness' of an image signal as

$$\alpha = \frac{|\{x \mid H(x) \neq 0\}|}{Max - Min + 1} \tag{1}$$

$$x \in [Min, Max] \in [0, 2^N) \in \text{integer}\ , \tag{2}$$

where $H(x)$ denotes the histogram of a pixel value x, and |X| denotes the total number of all the elements of a set X. Namely, the numerator of (1) indicates the number of non-zero bins of the histogram.

An HDR image tends to be 'sparse' (α<1) due to huge number of histogram bins (N=16, 32, e.g. N>8). It also occurs in LHR (low dynamic range) image as a result of pre-processing such as a histogram modification, a tone mapping, a Gamma correction and so on. In this report, we propose an encoding which utilizes this histogram sparseness.

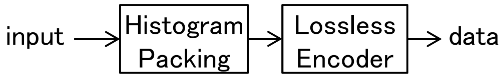

Fig.1 'Lossless' encoding for sparse histogram images.

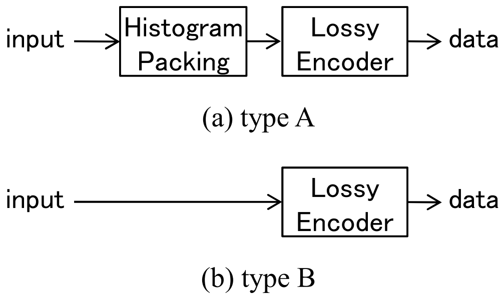

Fig.2 'Lossy' encoding for sparse histogram images.

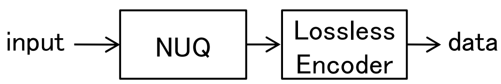

Fig.3 Proposed encoding for 'sparse histogram' images.

TABLE I Coding methods investigated in this report.

	Existing				Proposed
	JPEG-2K based		JPEG-LS based		(2K or LS based)
	type A	type B	type A	type B	
Rate Control	**fine**	**fine**	coarse	coarse	**fine**
S/N Ratio	low	middle	middle	**high**	**high**

B. Lossless Encoder and Histogram Packing

The histogram packing (HSP) introduced in [3] converts a 'sparse' histogram image into 'dense' one. It maps a sparse set of original pixel value $x(\mathbf{n})$ at location $\mathbf{n}$ to a dense set as

$$y(\mathbf{n}) = M \circ x(\mathbf{n}), \quad \mathbf{n} = [n_1, n_2] \tag{3}$$

where $M\circ$ denotes HSP as an operation. It holds

$$\begin{cases} \hat{x}(\mathbf{n}) - x(\mathbf{n}) = 0 \\ \hat{x}(\mathbf{n}) = M^{-1} \circ M \circ x(\mathbf{n}) \end{cases}, \tag{4}$$

namely, this procedure is lossless. Even though it does not change the 1st order entropy rate, it can reduce data volume when a lossless encoder is combined [4,5] as illustrated in Fig.1. However, it has been limited to 'lossless', and therefore the bit rate can't be controlled.

C. Lossy Encoder and Histogram Packing

To meet our purpose to construct a 'lossy' coding for histogram sparse images, a combination of HSP and a lossy encoder illustrated in Fig.2(a) is a candidate (type A). A simple lossy encoder in Fig.2(b) is another candidate (type B). We investigate the cases summarized in table I.

In Fig.2(a), when a transform is used in the lossy encoder like JPEG-2K [9], quantization noise generated inside the lossy encoder is magnified by the inverse HSP. It degrades quality of the reconstructed image. In this case, $y(\mathbf{n})$ in (3) is transformed by $T\circ$, and quantized with a step size $q_\mathbf{m}$ as

$$w(\mathbf{m}) = Round\left[q_\mathbf{m}^{-1} \cdot T \circ y(\mathbf{n})\right] = q_\mathbf{m}^{-1} \cdot T \circ y(\mathbf{n}) + e(\mathbf{m}) \tag{5}$$

where '$Round[p]$' denotes rounding to the integer nearest to p. It is reconstructed as

$$\hat{y}(\mathbf{n}) = T^{-1} \circ \left(q_\mathbf{m} \cdot w(\mathbf{m})\right) = T^{-1} \circ \left(T \circ y(\mathbf{n}) + q_\mathbf{m} \cdot e(\mathbf{m})\right) \tag{6}$$

where $e(\mathbf{m})$ denotes the quantization noise in transform domain. As a result, the reconstructed signal becomes

$$\begin{aligned} \hat{x}(\mathbf{n}) &= M^{-1} \circ \left(T^{-1} \circ \left(T \circ M \circ x(\mathbf{n}) + q_\mathbf{m} \cdot e(\mathbf{m})\right)\right) \\ &= M^{-1} \circ \left(M \circ x(\mathbf{n}) + T^{-1} \circ q_\mathbf{m} \cdot e(\mathbf{m})\right) \\ &= x(\mathbf{n}) + M^{-1} \circ T^{-1} \circ q_\mathbf{m} \cdot e(\mathbf{m}). \end{aligned} \tag{7}$$

This equation means that the probability density function of the quantization noise $e(\mathbf{m})$ is scattered by the inverse transform $T^{-1}\circ$, and magnified by the inverse HSP $M^{-1}\circ$.

For simplicity of our discussion, expressing the procedure of HSP as

$$M \circ x = \gamma_x \cdot x, \quad M^{-1} \circ x = \gamma_x^{-1} \cdot x \tag{8}$$

with a scaling factor γ_x, the procedure in (7) is expressed as

$$\hat{x}(\mathbf{n}) = x(\mathbf{n}) + \gamma_x^{-1} \cdot T^{-1} \circ q_\mathbf{m} \cdot e(\mathbf{m}). \tag{9}$$

Since $\gamma_x<1$ for a sparse histogram image, high quality of reconstructed images can't be obtained. In this case, S/N ratio becomes 'low' as indicated in the table I.

Similarly, when a near lossless prediction is used in Fig.2(a) like JPEG-LS [10], a reconstructed image becomes

$$\hat{x}(\mathbf{n}) = x(\mathbf{n}) + \gamma_x^{-1} \cdot q_\mathbf{m} \cdot e'(\mathbf{n}) \tag{10}$$

where $e'(\mathbf{n})$ denotes the quantization noise in spatial domain. It should be noted that the step size $q_\mathbf{m}$ must be an integer since all the pixel values are expressed as integers. Therefore, the rate control is limited to 'coarse' as summarized in the table.

As mentioned above, type A in Fig.2(a) is not suitable for lossy coding for sparse histogram images since the inverse procedure of HSP magnifies quantization noise generated in the lossy coding (JPEG-2K [10] or LS [9]). On the contrary, type B in Fig.2(b) does not have such noise-magnification problem. However, it can't utilize sparseness in histogram of input images. We experimentally investigate performance of these existing methods in the rate-distortion curve.

III. Proposed Method

We introduce a new lossy coding for sparse histogram images. It can attain high image quality and fine rate control.

A. Lossless Encoder and Non-Uniform Quantization

Fig.3 illustrates the proposed encoding approach. We introduce a non-uniform quantization (NUQ) for histogram sparse images, and combine it with a normal lossless encoder. In this case, the reconstructed image is described as

$$\hat{x}(\mathbf{n}) = P^{-1} \circ T^{-1} \circ \left(T \circ P \circ x(\mathbf{n})\right) \tag{11}$$

where

$$\begin{cases} T^{-1} \circ T \circ y(\mathbf{n}) - y(\mathbf{n}) = 0 \\ P^{-1} \circ P \circ x(\mathbf{n}) - x(\mathbf{n}) = e''(\mathbf{n}) \end{cases} \tag{12}$$

and $P\circ$ denotes NUQ. Note that the noise $e''(\mathbf{n})$ is generated in spatial domain by NUQ. Therefore, the noise is not magnified by the inverse HSP, and high quality images can be reconstructed. Procedure of NUQ is detailed as below.

B. Non-Uniform Quantization for Histogram Sparse Signals

NUQ reduces 2^N kinds of tones for a pixel value x to L tones ($2^N > L$) utilizing 'sparseness' of the histogram $H(x)$. Forward mapping and backward mapping are performed with tables Q and R as

$$\begin{cases} y(\mathbf{n}) = \ \ P \circ x(\mathbf{n}) = Q\left(x(\mathbf{n})\right) \\ \hat{x}(\mathbf{n}) = P^{-1} \circ y(\mathbf{n}) = R\left(y(\mathbf{n})\right) \end{cases}. \tag{13}$$

Note that these tables are included into a bit stream.

Firstly, prepare a histogram $H(k)$ of integer pixel values $k \in [0,\, 2^N)$. We consider the case where not all the 2^N bins but only N_e bins are used (NZ: non zero) in the input image ($N_e < 2^N$). Fig.4 illustrates an example for N_e=11 and 2^N=16.

Secondly, reduce the number of bins from 2^N to L, unifying a few neighboring 'NZ' bins into one class, ignoring '0' bins so that the sparseness is fully utilized for reduction of the range of pixel values. For example, when each of $s = \lfloor N_e / L \rfloor$ neighboring NZ bins are unified into one class, there are $N_e - sL = N_h$ remaining NZ bins. Therefore, we unify s+1 neighboring NZ bins into one class for N_h classes (N_h=3 in Fig.4), and unify s neighboring NZ bins into one class for L-N_h classes. As a result, N_e bins are unified into L classes in total.

Finally, calculate the tables Q and R in (13) as below.

$$Q(m_i) = \begin{cases} \left\lfloor \dfrac{i}{s+1} \right\rfloor & , \quad i \in \left[\, 0,\, N_h(s+1)\, \right) \\ \left\lfloor \dfrac{i - N_h(s+1)}{s} \right\rfloor + N_h, & i \in \left[N_h(s+1),\, N_e\right) \end{cases} \tag{14}$$

and

$$R(n) = \begin{cases} \dfrac{\sum_{i=(s+1)n}^{(s+1)n+s} H(m_i) \cdot m_i}{\sum_{i=(s+1)n}^{(s+1)n+s} H(m_i)} & , \quad n \in \left[0,\, N_h\right) \\ \dfrac{\sum_{i=sn+N_h}^{sn+N_h+s-1} H(m_i) \cdot m_i}{\sum_{i=sn+N_h}^{sn+N_h+s-1} H(m_i)} & , \quad n \in \left[N_h,\, L\right) \end{cases} \tag{15}$$

where

$$m_i = \left\{ k \in [0,\, 2^N) \;\middle|\; H(k) \neq 0 \right\},\ i \in [0,\, N_e) \tag{16}$$

indicates location of NZ bins in the original histogram.

According to the table Q, the original 2^N pixel tones are quantized into L tones utilizing the histogram sparseness in the encoder. The right-hand side of (15) indicates the centroid of all the bins in a class. It is rounded and used in the decoder. This procedure is simpler than the Max-Lloyd quantization.

←——— INPUT ———→

k =	00	01	02	03	04	05	06	07	08	09	10	11	12	13	14	15	← 2^N-1
H(k) =	NZ	NZ	NZ	NZ	NZ	NZ	0	0	NZ	0	0	0	NZ	NZ	NZ	NZ	Histogram
i =	00	01	02	03	04	05	. .	. .	06	. .	. .	. .	07	08	09	10	← N_e-1
m_i =	00	01	02	03	04	05	. .	. .	08	. .	. .	. .	12	13	14	15	Location
Q(m_i) =	00	00	01	01	02	02	. .	. .	03	. .	. .	. .	04	05	06	07	← L−1
n =	00	—	01	—	02	—	. .	. .	03	. .	. .	. .	04	05	06	07	← L−1
R(n) =	01	—	02	—	04	—	. .	. .	08	. .	. .	. .	12	13	14	15	Represent

(NZ = Non Zero)

←s+1→ ←s+1→ ←s+1→ ← $N_h(s+1)$ → ; ←→ s=1 ; ←→ s, s, s, s=1, N_h=3

← N_h classes → ← $L-N_h$ classes →

Fig.4 Example of the non-uniform quantization (NUQ) procedure for a sparse histogram $H(k)$.

IV. EXPERIMENTS

The proposed method in Fig.3 is compared to the existing methods in Fig.2 in the rate-distortion curve for high bit depth two test images (Thumb nails are indicated in Fig.5).

(a) 'Cafe' (4,992×3,328 pxl) (b) 'CT' (512×512 pxl)

Fig.5 Test images.

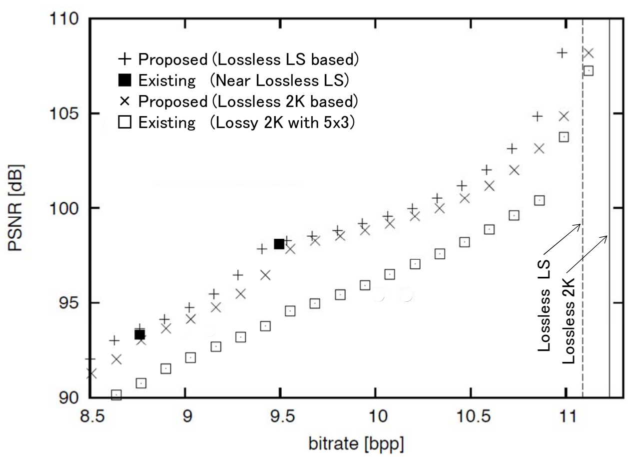

Fig.6 Rate distortion curves of a 'dense' image 'Cafe'. Bit depth N=16, N_e=64,486, Sparseness α=0.98

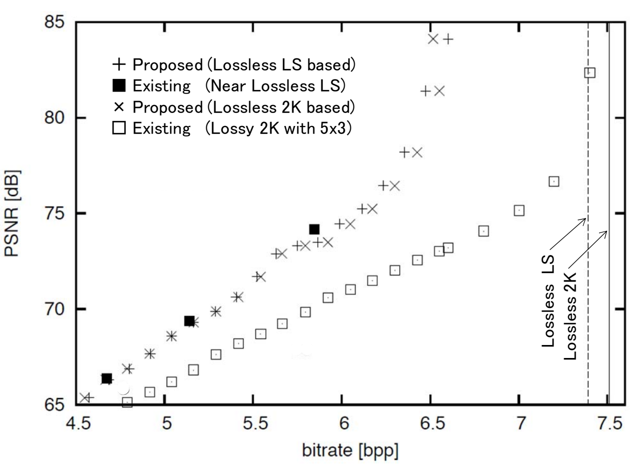

Fig.7 Rate distortion curves of a 'sparse' image 'CT'. Bit depth N=12, N_e=1,853, Sparseness α=0.45

Fig.6 and Fig.7 compare the existing method and the proposed method for 'Cafe' and 'CT' in Fig.5 respectively. In these figures, the proposed method in Fig.3 based on the lossless JPEG-LS was observed to be the best, followed by the proposed method based on the lossless JPEG-2K.

The label 'existing method (Near Losslesss LS)' indicates the method in Fig.2(b). This is 'LS based type B' in table I. It is confirmed that it attains 'high' SNR almost the same level of the proposed method. However, its rate control is 'coarse' as summarized in the table.

The label 'existing method (Lossy 2K with 5x3)' indicates the method in Fig.2(b). This is based on the reversible 5x3 discrete wavelet transform and the bit-truncation in JPEG 2000. It was labeled as '2K based type B' in table I. It is confirmed that its SNR is worse than the proposed method. However, its rate control is 'fine' as summarized in the table.

According to Fig.7 for a 'sparse' image, it is observed that the sparseness is utilized and therefore the bit rate is reduced by the proposed method. It becomes obvious when it is compared to the lossy JPEG 2K. Note that we have already confirmed that the existing method 'type A' in Fig.2(a) is much worse that the methods in Fig.6 and Fig.7.

V. CONCLUSIONS

In this report, we introduced a non-uniform quantization for histogram sparse images, and combined it with a lossless encoder. It was confirmed that the proposed method attains higher SNR than the existing lossy JPEG-2000, and also finer rate control than the existing near lossless JPEG-LS.

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