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THREE DIMENSIONAL DISCRETE WAVELET TRANSFORM WITH REDUCED NUMBER OF LIFTING STEPS

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ABSTRACT

This report reduces the total number of lifting steps in a three-dimensional (3D) double lifting discrete wavelet transform (DWT), which has been widely applied for analyzing volumetric medical images. The lifting steps are necessary components in a DWT. Since calculation in a lifting step must wait for a result of former step, cascading many lifting steps brings about increase of delay from input to output. We decrease the total number of lifting steps introducing 3D memory accessing for the implementation of low delay 3D DWT. We also maintain compatibility with the conventional 5/3 DWT defined by JPEG 2000 international standard for utilization of its software and hardware resources. Finally, the total number of lifting steps and rounding operations were reduced to 67 % and 33 %, respectively. It was observed that total amount of errors due to rounding operations in the lifting steps was also reduced.

***Index Terms*—** wavelet, 3D, coding, medical

1. INTRODUCTION

Recently, three dimensional (3D) discrete wavelet transform (DWT) has been applied to compressing huge amount of data size of volumetric medical images [1]. It has been also applied to hyper spectral images [2]. To construct a 3D DWT, it is convenient to cascade conventional one dimensional (1D) DWTs three times along with vertical, horizontal and one more axis. However it takes long time for total processing from input to output (delay). In this report, we propose a low delay 3D DWT compatible with the conventional 5/3 DWT defined by JPEG 2000 international standard [3].

In case of two dimensional (2D) signal processing, a 1D DWT is applied vertically (or horizontally) to a 2D image signal, and then horizontally (or vertically). Since this processing is expressed as a product of vertical 1D transfer function and horizontal one, it is referred to 'separable' 2D DWT. Although it is advantageous that its memory accessing is limited to 1D (line memory) [4], it requires many 1D processing steps in cascade resulting in long delay from input to output. Flexibility of directional 2D filtering is also restricted. Therefore several 'non-separable' 2D DWTs have been reported to make it free from restrictions under the 'separable' 2D structure [5-13].

A non-separable 2D structure composed of double lifting DWTs (5/3 filter bank) defined by JPEG 2000 for lossless coding is reported in [8,9]. Introducing 2D memory accessing, delay from input to output is decreased as the total number of lifting steps is reduced. The theory is extended to the quadruple lifting DWT (9/7 filter bank) in [10]. Its lifting steps were furthermore reduced in [11]. Its performance was investigated from various points of view and filter coefficients were optimized in [12].

In this report, we extend the previous discussions for 2D to 3D. Introducing 3D memory accessing, we reduce the total number of lifting steps in the 3D DWT compatible with the conventional double lifting DWT. It is equivalent to replace the conventional 'separable' 3D transfer function with the 'non-separable' 3D function. In our experiments, it is indicated that the total number of lifting steps is decreased, as well as the total amount of rounding errors.

2. PREVIOUS WORKS

2.1. One Dimensional (1D) Lifting Wavelet

Fig.1 illustrates the 1D double lifting DWT (5/3 filter bank) in JPEG 2000. It classifies an L length input sequence $x(m)$, $m=0,1,\cdots,L-1$, into two classes $a(n)$ and $b(n)$ as

$$a(n)=x(2n),\quad b(n)=x(2n+1),\quad n\in[0,L/2)\,. \qquad (1)$$

These are converted to low frequency band signal $a'(n)$ and high band signal $b'(n)$ through two (double) lifting steps as

$$\begin{cases} b'(n)=b(n)+R[H_\alpha * a(n)] & \cdots\ 1st\ step \\ a'(n)=a(n)+R[H_\beta * b'(n)] & \cdots 2nd\ step \end{cases} \qquad (2)$$

where

$$\begin{cases} H_\alpha * a(n) = \alpha\big(a(n)+a(n+1)\big) \\ H_\beta * b'(n)=\beta\big(b'(n)+b'(n-1)\big) \end{cases} \qquad (3)$$

and $R[p]$ denotes an integer close to p (rounding operation). A set of filter coefficients is defined as $(\alpha, \beta)=(-1/2, 1/4)$ in JPEG 2000 international standard.

Since the 2nd lifting step must wait for a result of the 1st lifting step in (2), overall delay from input to output crucially depends on the total number of these lifting steps in the DWT. In addition, total amount of rounding noise depends on the total number of rounding operations $R[\ ]$ in (2). Therefore, in this report, we reduce these numbers for implementation of a low delay 3D DWT with fine signal quality.

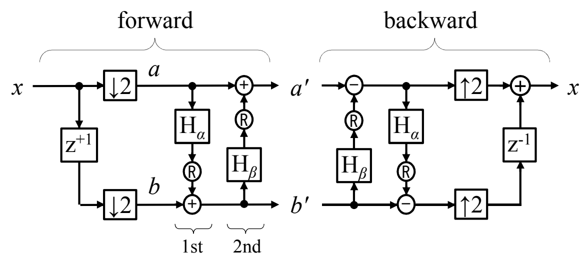


Fig.1 Lifting structure of the 1D double lifting DWT.

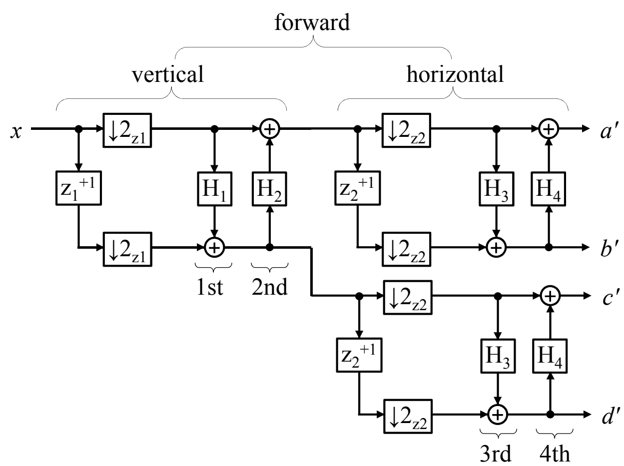


Fig.2 Separable 2D structure of the double lifting DWT.

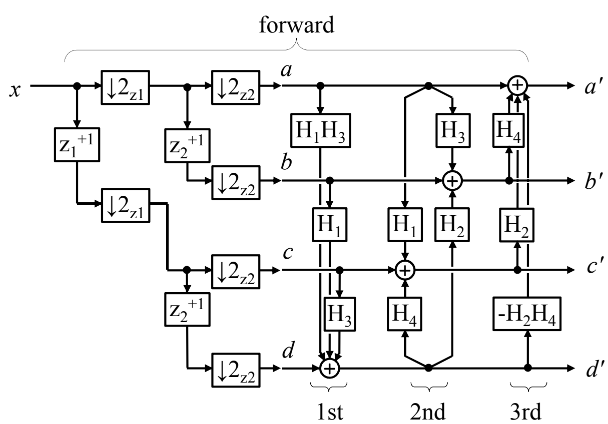


Fig.3 Non-separable 2D structure of the double lifting DWT.

2.2. Two Dimensional (2D) Lifting Wavelet

Fig.2 illustrates the 'separable' 2D structure. The 1D double lifting DWT is applied vertically, and then horizontally. Note that rounding operations just before addition are omitted in the figure for simplification without loss of generality. As a result, we have four band signals a', b', c' and d'. It contains 1st, 2nd, 3rd and 4th lifting steps.

Fig.3 illustrates the 'non-separable' 2D double lifting DWT with reduced number of lifting steps [8,9]. After classifying the input signal x into four classes a, b, c and d, four frequency band signals a', b', c' and d' are produced though 1st, 2nd and 3rd lifting steps. Note that the total number of lifting steps is reduced from 4 in Fig.2 to 3 in Fig.3. In this report, we extend this theory for 2D to 3D.

2.3. Separable 3D Wavelet (Existing Method)

Fig.4 illustrates a 'separable' 3D structure. The 1D double lifting DWT is applied to $x(m_1,m_2,m_3)$ for $m_1 \in [0,L_1)$, $m_2 \in [0,L_2)$, $m_3 \in [0,L_3)$ to split into eight band signals. After classifying the input signal into eight groups:

$$\begin{cases} a(\mathbf{n}) = x(2n_1, 2n_2, 2n_3),\ b(\mathbf{n}) = x(2n_1, 2n_2, 2n_3+1) \\ c(\mathbf{n}) = x(2n_1, 2n_2+1, 2n_3),\ d(\mathbf{n}) = x(2n_1, 2n_2+1, 2n_3+1) \\ e(\mathbf{n}) = x(2n_1+1, 2n_2, 2n_3),\ f(\mathbf{n}) = x(2n_1+1, 2n_2, 2n_3+1) \\ g(\mathbf{n}) = x(2n_1+1, 2n_2+1, 2n_3),\ h(\mathbf{n}) = x(2n_1+1, 2n_2+1, 2n_3+1) \end{cases} \tag{4}$$

for $(\mathbf{n})=(n_1,n_2,n_3)$, $n_1 \in [0,L_1/2)$, $n_2 \in [0,L_2/2)$, $n_3 \in [0,L_3/2)$, the 1st and the 2nd lifting steps produce

$$\begin{cases} p''(\mathbf{n}) = p(\mathbf{n}) + R[H_1 * q(\mathbf{n})] \\ q''(\mathbf{n}) = q(\mathbf{n}) + R[H_2 * p''(\mathbf{n})] \end{cases} \tag{5}$$

for $(p,q)=\{(e,a), (f,b), (g,c), (h,d)\}$ where the filters

$$\begin{cases} H_1 * p(\mathbf{n}) = \alpha\big(p(\mathbf{n}) + p(n_1+1, n_2, n_3)\big) \\ H_2 * q(\mathbf{n}) = \beta\big(q(\mathbf{n}) + q(n_1-1, n_2, n_3)\big) \end{cases} \tag{6}$$

operate on variable n_1. Similarly, (H_3,H_4) and (H_5,H_6) in Fig.4 operate on variables n_2 and n_3, respectively. This existing 'separable' 3D DWT has six lifting steps.

3. NON-SEPARABLE 3D WAVELET

Fig.5 illustrates our proposal. In the 1st lifting step, h' is calculated as

$$\begin{aligned} h'(\mathbf{n}) = h(\mathbf{n}) + R[&H_1H_3H_5 * a(\mathbf{n}) + H_1H_3 * b(\mathbf{n}) \\ &+ H_1H_5 * c(\mathbf{n}) + H_1 * d(\mathbf{n}) + H_3H_5 * e(\mathbf{n}) \\ &+ H_3 * f(\mathbf{n}) + H_5 * g(\mathbf{n})] \end{aligned} \tag{7}$$

where $H_1H_3H_5$ denotes cascading H_1, H_3 and H_5. This is a 3D memory accessing defined as

$$
\begin{aligned}
&H_1 H_3 H_5 * a(\mathbf{n}) = \\
&+\alpha^3 \{ a(n_1, n_2, n_3) + a(n_1+1, n_2, n_3) \\
&+ a(n_1, n_2+1, n_3) + a(n_1+1, n_2+1, n_3) \\
&+ a(n_1, n_2, n_3+1) + a(n_1+1, n_2, n_3+1) \\
&+ a(n_1, n_2+1, n_3+1) + a(n_1+1, n_2+1, n_3+1) \}
\end{aligned}
\tag{8}
$$

After waiting for calculation result $h'(\mathbf{n})$ in the 1st step, the 2nd lifting step produces

$$
\begin{cases}
g' = g + R[H_1 H_3 * a + H_1 * c + H_3 * e + H_6 * h'] \\
f' = f + R[H_1 H_5 * a + H_1 * b + H_5 * e + H_4 * h'] \\
d' = d + R[H_3 H_5 * a + H_3 * b + H_5 * c + H_2 * h']
\end{cases}
\tag{9}
$$

where $(\mathbf{n})$ is omitted. Note that the three equations in (9) can be done simultaneously. Similarly, the 3rd and the 4th lifting steps calculate

$$
\begin{cases}
b' = b + R[-H_2 H_4 * h' + H_2 * f' + H_4 * d' + H_5 * a] \\
c' = c + R[-H_2 H_6 * h' + H_2 * g' + H_6 * d' + H_3 * a] \\
e' = e + R[-H_4 H_6 * h' + H_4 * g' + H_6 * f' + H_1 * a]
\end{cases}
\tag{10}
$$

and

$$
\begin{aligned}
a' = a + R[&H_2 H_4 H_6 * h' - H_2 H_4 * g' - H_2 H_6 * f' \\
&+ H_2 * e' - H_4 * H_6 * d' + H_4 * c' + H_6 * b']
\end{aligned}
\tag{11}
$$

respectively.

Finally, the total number of lifting steps is reduced from 6 in the existing 'separable' 3D structure to 4 (66.7 [%]) in the proposed 'non-separable' 3D structure in Fig.5. It is guaranteed that the output band signals in the proposed DWT are exactly the same as those of the existing DWT in case of rounding operations $R[\ ]$ are not embedded. In this sense, our DWT has full compatibility with the existing DWT under a given filter set (H_α, H_β) in (2).

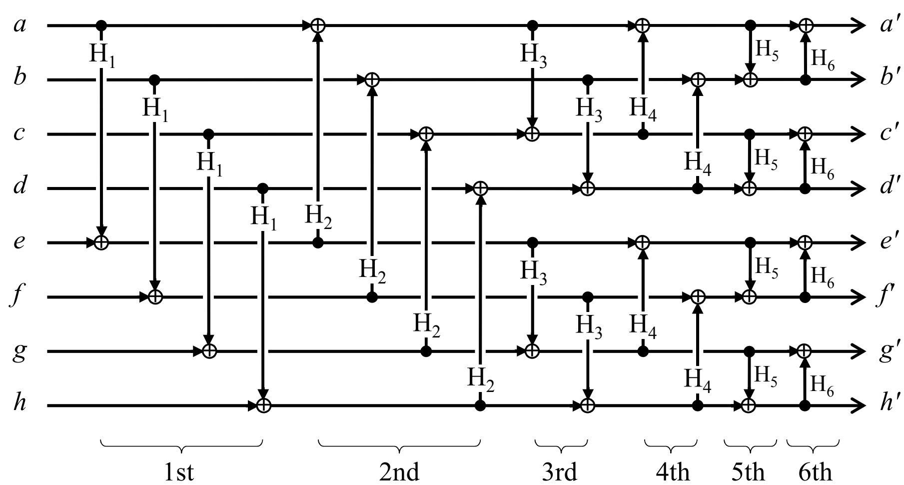


Fig.4 Separable 3D structure of the double lifting DWT (existing method).

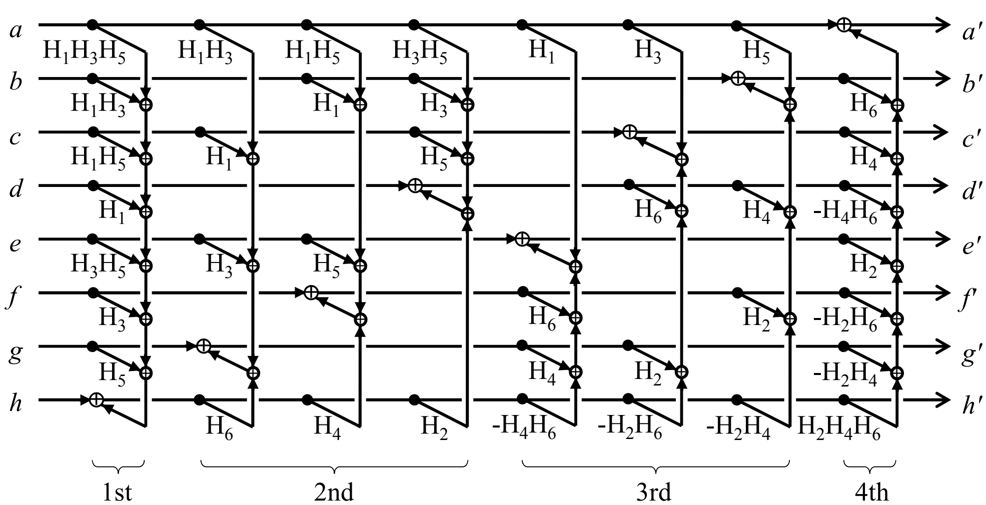


Fig.5 Non-separable 3D structure of the double lifting DWT (proposed method).

4. RESULTS

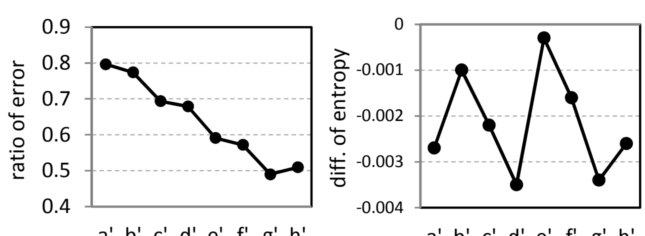
Table I compares the existing 'separable' 3D DWT and the proposed 'non-separable' 3D DWT. Obviously, as a result of our proposal, the total number of lifting steps is reduced from 6 to 4. Due to this reduction, it is expected to the implementation of a low delay 3D DWT in the near future.

For lossless coding, a rounding operation is inserted between the filter and the adder in each lifting step. In this case, the number of rounding operations is 24 in the existing method. On the contrary, the proposed DWT in Fig.5 has only 8 as indicated in (7), (9), (10) and (11). Consequently, the number of rounding operations is also reduced from 24 to 8 (33.3 %).

Since fewer rounding operations does not always imply less rounding errors in total [12,13], we evaluated total amount of rounding errors contained in the output frequency band signals. Difference between output from DWT without rounding and that with rounding is defined as the error. Fig.6 (a) indicates ratio (=existing/proposed) of the standard deviation (SD) of the rounding errors. Total amount of the rounding errors is reduced to 63 [%] in average over all the bands. Fig.6 (b) indicates difference (=existing-proposed) of the entropy rate. No significant difference was observed. It was clearly indicated that the proposed DWT significantly reduces rounding errors with satisfactory compatibility.

Table I Comparison between the two DWTs.

	rounding operations	lifting steps
separable	24 (100%)	6 (100%)
non separable	8 (33.3%)	4 (66.7%)



(a) rounding error (b) entropy rate

Fig.6 Comparison between the two methods.

4. CONCLUSIONS

A non-separable 3D DWT compatible with the existing separable 3D 'double' lifting DWT was proposed. Introducing 3D direct memory accessing, the total number of lifting steps was reduced to 67 [%]. The total number of rounding operations is also reduced to 33 [%]. It is advantages for implementation that our DWT has compatibility with a DWT module widely used as an international standard.

Since our DWT has less lifting steps and less rounding operations, it is expected to contribute to the implementation of a low delay 3D DWT with fine signal quality for compression of medical volumetric data in the near future.

5. REFERENCES

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