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Analytical Evaluation of Integer DCT

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Abstract--- **This report analytically evaluates some previously proposed integer DCT algorithms in respect of compatibility with the normal DCT, the rounding error peculiar to the integer DCT, performance of lossless coding and lossy coding, respectively. Under the assumption that the image signal's characteristics is approximated to AR(1) model, optimum word length is assigned to each of the multipliers and error due to the finite word length expression of the multipliers is evaluated.**

I. INTRODUCTION

Discrete cosine transform (DCT) is one of the most popular transforms for lossy coding of still image or video signals since the international standard JPEG and MPEG have adopted the DCT. However, the DCT has not been used for lossless coding since it outputs real numbers in general.

The next international standard JPEG-2000 replaced the DCT by the integer wavelet transform (WT) which contains the rounding operations in the lifting structure [1]. As a result, the lossless coding with the transform became possible.

The integer DCT, similar to the integer WT, is also attractive since it can be used for not only lossless coding but also lossy coding compatible to the world widely prevailed conventional DCT.

So far, various types of the integer DCT algorithms are proposed [2-8]. It is also extended to more general types of the integer DCT [9,10]. This report aims at analytically evaluating previously proposed integer DCT algorithms in respect of compatibility with the conventional DCT, variance of the rounding error peculiar to the integer DCT, performance of lossless coding and lossy coding, respectively.

In addition, optimum word length is assigned to each of the multipliers in the integer DCT under the assumption that the image signal's characteristics is approximated to AR(1) model [11]. The error due to this finite word length expression of the multipliers is evaluated.

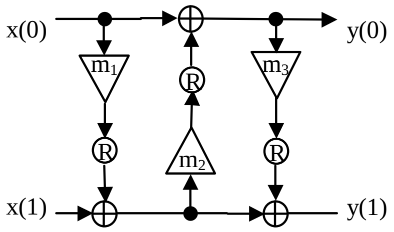

Fig.1 Integer rotation (IR) transform.

II. INTEGER DCT ALGORITHMS

All of the integer DCT algorithms compared in this report [2-8] are compatible to the conventional DCT defined by

$$y(k) = \sum_{n=0}^{7} a_{k,n} \cdot x(n), \quad k = 0,1,\cdots,7 \tag{1}$$

where

$$a_{k,n} = \begin{cases} \frac{1}{2}\cos\frac{(2k+1)n\pi}{16}, & k = 1,2,\cdots,7 \\ \frac{1}{\sqrt{8}}\cos\frac{n\pi}{16}, & k = 0 \end{cases} .$$

A. *The Integer DCT Algorithms*

All the integer DCT in this report contains the integer rotation (IR) transform illustrated in figure 1 which includes the rounding operations "R" and three multipliers "m_1", "m_2" and "m_3".

Figure 2 indicates Fukuma's integer DCT in [3,4] and it contains three integer Hadamard transform IH which does not contain multiplier and rounding operation.

Figure 3 illustrates Chen's DCT in [2] whose rotation transform is replaced by IR.

Figure 4 illustrates Soontorn's integer DCT in [5]. H_w and B in the figure stand for 8-point Walsh-Hadamard transform and permutation matrix, respectively. Both of them are composed of no rounding operation and no multiplier. Notice that it requires scaling with $8^{1/2}$ to its output signal.

Other integer DCT algorithms are Somchart's one in [8] and Philips's one in [6,7]. The latter employs modulo calculation and does not use multiplier.

B. *The Number of Rounding Operations*

The IR transform has three rounding operations. The IH transform has two. Therefore, Fukuma's integer DCT with five IR and three IH contains 3*5+2*3=21 rounding operations in total. Chen's integer DCT with 13 IR and Soontorn's with five IR contains 3*13=39 and 3*5=15 rounding operations, respectively.

When the backward integer DCT is connected to the forward conventional DCT, SNR degradation is observed on the decoded image signal. This rounding error is evaluated in section III.

C. *The Number of Multipliers*

The IR transform has three multipliers. Therefore, Fukuma's integer DCT with five IR contains 3*5=15 multipliers in total. Chen's integer DCT with 13 IR and Soontorn's with five IR contains 3*13=39 and 3*5=15 multipliers, respectively.

Word length of the multiplier, for example in table 1, is truncated into finite length in practice. In addition, the shorter the length is, the smaller the LSI circuit becomes resulting in lesser power consumption. Finite word length expression of the multiplier brings about SNR degradation of the decoded image signal. This error is evaluated in section III under the optimum word length assignment described next.

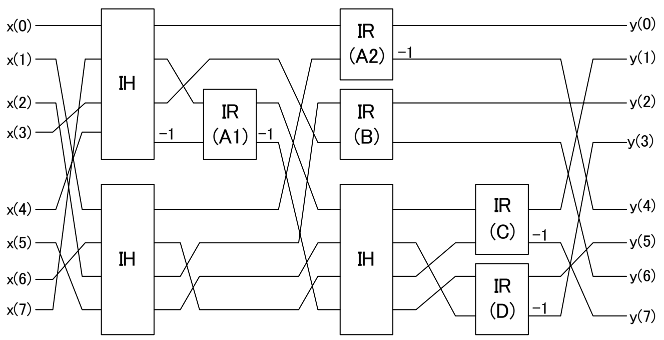

Fig.2 Fukuma's DCT [3,4].

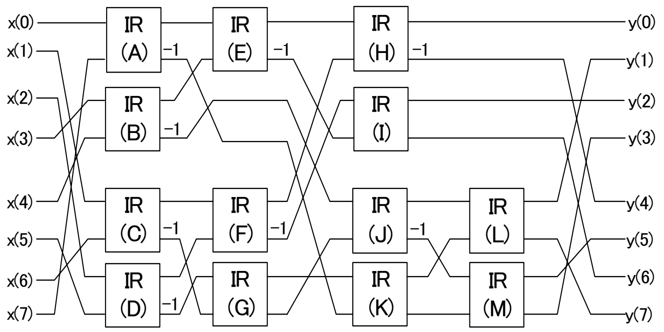

Fig.3 Modified Chen's DCT [2].

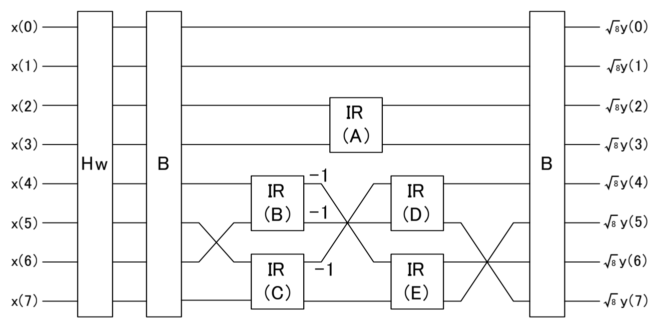

Fig.4 Soontorn's DCT [5].

D. *Optimum Word Length Assignment*

The optimum word length assignment to each of the multipliers in the integer DCT is summarized as below under the assumption that the image signal's characteristics is approximated to AR(1) model with correlation ρ [11].

For example, we assume that an input signal is divided into blocks and the forward integer DCT with infinite word length multipliers is applied to each block. After this, the backward integer DCT in which all of multipliers' word length are infinite except only one multiplier h_k, $k \in \{0,1,\cdots,K-1\}$. In this case, the decoded signal contains error due to finite word length expression of only one multiplier.

Denoting input vector to the forward DCT and output vector from the backward transform in a block as $\mathbf{X} = [\,x(0)\cdots x(7)\,]^T$ and $\hat{\mathbf{X}} = [\,\hat{x}(0)\cdots \hat{x}(7)\,]^T$ respectively, the error is expressed by

$$\Delta\sigma_k^2 = E\left[\left(\hat{\mathbf{X}}-\mathbf{X}\right)^T\left(\hat{\mathbf{X}}-\mathbf{X}\right)\right] \tag{2}$$

where E[ ] and "T" denote ensemble average and transpose of a matrix. This error is due to displacement of finite word length expression of a multiplier h_k described by

$$\hat{h}_k = h_k + \Delta h_k \tag{3}$$

for $k \in \{0,1,\cdots,K-1\}$ where Δh_k is related to the word length W_k, namely $\Delta h_k \propto 2^{-W_k}$. Then, the sensitivity is defined by

$$S_k = \frac{\Delta\sigma_k}{\Delta h_k} \quad . \tag{4}$$

The optimum word length assignment is solved as a solution of the least square method under a constrain, namely,

$$\text{Minimize} \qquad \sigma_{total}^2 = \sum_{k=0}^{K-1} \Delta\sigma_k^2$$

$$\text{under a given} \quad \overline{W} = \frac{1}{K}\sum_{k=0}^{K-1} W_k \quad .$$

As a result, the optimum word length W_k for a multiplier h_k is given by

$$W_k = SR_k + \overline{W} \tag{5}$$

where

$$SR_k = \log_2 \frac{S_k}{\overline{S}}$$

$$\overline{S} = \prod_{k=0}^{K-1} \sqrt[K]{S_k} \; .$$

SR_k indicates the normalized sensitivity to the word length truncation of a multiplier h_k.

III. EVALUATION AND COMPARISON

A. *Lossless Coding*

Performance of the integer DCT in lossless coding mode was evaluated for some standard image signals. The first order entropy rate of the transformed signal is indicated in figure 5.

Fukuma' algorithm and Chen's one are the best, followed by Somchart's integer DCT and Philips' one. In case of Soontorn's integer DCT, it can be considered that the scaling factor of $8^{1/2}$ adversely affects on lossless coding performance. It requires three bit augmentation to the signal when the transform is applied twice, namely vertically and horizontally.

B. *Lossy Coding*

Performance of the integer DCT in lossless coding mode was evaluated for AR(1) model with correlation coefficient ρ =0.95. No difference between the conventional DCT and various integer DCT is observed in the rate-distortion curves, entropy rate versus peak-signal to noise ratio (PSNR), in figure 6. As a result, compatibility of the integer DCT with the conventional DCT on lossy coding was confirmed.

C. *Compatibility with the conventional DCT*

Table 2 indicates root mean square error (RMSE) between input AR(1) model signal and decoded signal. PSNR is also accompanied. In this experiment, the signal is encoded with the conventional forward DCT and decoded with each of the backward integer DCT. No quantization between forward and backward transform was performed. The error is confirmed to be negligible for all algorithms.

D. *Error due to Rounding Operations*

Table 3 indicates RMSE and PSNR of decoded signals for the AR(1) model. The signal is encoded with the forward integer DCT and decoded with each of the backward integer DCT. However, all the rounding operations are excluded from the forward transform. This is equivalent to connecting a conventional forward DCT and the integer backward DCT. In this case, we can observe only rounding errors.

Fukuma's algorithm has the least RMSE and also the number of multipliers. Chen's one is the most. There is a positive correlation between RMSE in table 3 and the number of multipliers in table 4.

E. *Error due to Finite Word Length Expression*

Figure 7 illustrates PSNR of the decoded signal that contains error due to word length truncation versus the average word length $\overline{w}$ in equation (5). All the rounding operations were excluded from the algorithms in figure 2, 3, 4 and no quantization is applied between forward and backward transforms. As a result, it is observed that Fukuma's and Soontorn's are better than Chen's algorithm.

Table 1 Multiplier coefficient values of **IR** transforms in the Fukuma's integer DCT algorithm.

$m_1(A1)$	h_0	$1-\sqrt{2}$	$m_1(C)$	h_9	$\frac{1-\cos(3\pi/16)}{\sin(\pi/16)}$
$m_2(A1)$	h_1	$\frac{1}{\sqrt{2}}$	$m_2(C)$	h_{10}	$-\sin(3\pi/16)$
$m_3(A1)$	h_2	$1-\sqrt{2}$	$m_3(C)$	h_{11}	$\frac{1-\cos(3\pi/16)}{\sin(\pi/16)}$
$m_1(A2)$	h_3	$1-\sqrt{2}$	$m_1(D)$	h_{12}	$\frac{\cos(\pi/16)-1}{\sin(\pi/16)}$
$m_2(A2)$	h_4	$\frac{1}{\sqrt{2}}$	$m_2(D)$	h_{13}	$\sin(\pi/16)$
$m_3(A2)$	h_5	$1-\sqrt{2}$	$m_3(D)$	h_{14}	$\frac{\cos(\pi/16)-1}{\sin(\pi/16)}$
$m_1(B)$	h_6	$\frac{\sin(\pi/8)-1}{\cos(\pi/8)}$			
$m_2(B)$	h_7	$\cos(\pi/8)$			
$m_3(B)$	h_8	$\frac{\cos(3\pi/8)-1}{\cos(\pi/8)}$			

Table 2 Compatibility with the conventional DCT.

Fukuma's	Chen's	Soontorn's
1.39×10^{-14} (325 dB)	1.73×10^{-14} (323 dB)	1.46×10^{-14} (325 dB)

Table 3 Rounding error in the integer DCT.

Fukuma's	Chen's	Soontorn's
0.55 (53 dB)	0.66 (52 dB)	0.42 (56 dB)

Table 4 The number of rounding operations.

Fukuma's	Chen's	Soontorn's
21	39	15

IV. CONCLUSIONS

Some previously proposed integer DCT algorithms were compared with various criteria. In respect of compatibility with the normal DCT, there is no difference. As for the rounding error, Fukuma's algorithm is the best. In lossless coding, Fukuma's and Somchart's are the best. In lossy coding, there is no difference. Regarding finite word length expression of multipliers, Soontorn's integer DCT is the best. Other DCT algorithms will be included in the future.

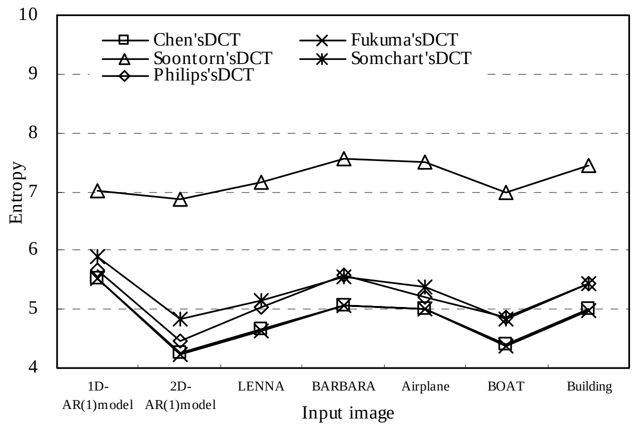

Fig.5 Performance in lossless coding mode.

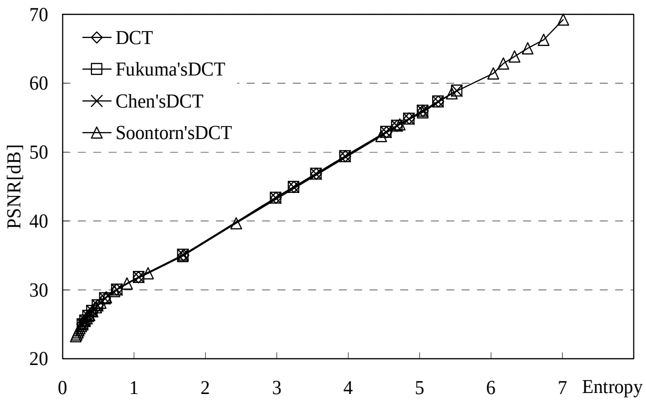

Fig.6 Performance in lossy coding mode.

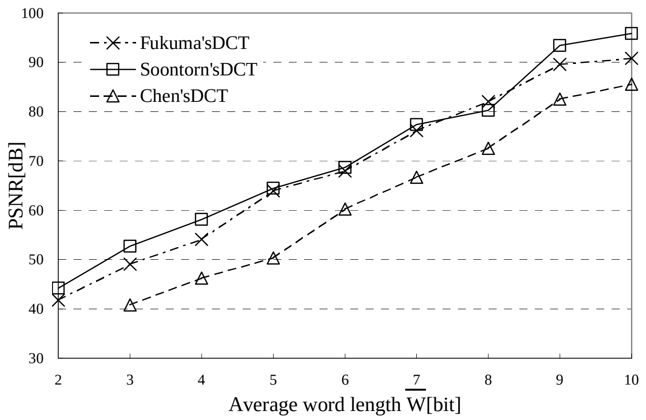

Fig.7 Error due to the finite word length expression of multipliers are plotted for average word length.

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