Abstract—This report introduces an error equalization to increase quality of low dynamic range (LDR) images in a backward compatible high dynamic range (HDR) image coding. To utilize a currently standard source coding algorithm such as the JPEG encoder, a data compression friendly mapping such as the power function is applied before compression. In a receiver side, the decoded image is once again mapped based on a visually proper function such as the Hill function to generate the LDR image. In this report, we investigate how the errors added in the data compression process are magnified through the inverse power function and the Hill function, and show that the errors in dark pixels of the LDR image are extremely high. Based on this, we equalize probability density function of the error in the LDR image to increase its quality. As a result of experiments, it was observed that the PSNR of the LDR was increased by approximately 3 to 4 [dB] in the rate distortion curves.

I. INTRODUCTION

Over the past few years, high dynamic range (HDR) images have been applied to various advanced image processing systems since HDR contains more information than currently standard low dynamic (LDR) range images [1,2]. In general, range of pixel values in an HDR image is reduced by tone mapping (TM) so that the image can be displayed with currently standard devices.

So far, various types of functions have been utilized as TM such as a power function for Gamma correction [3] and a logarithmic function for range reduction [4]. Especially, the Hill function has been widely applied as TM since it meets human visual property [5,6]. In this report, we consider not only visual suitability, but also data compression appropriateness in selecting the mapping function.

We consider the case in which 1) range of pixel values of the original HDR image is normalized so that it meets the currently standard encoder such as JPEG, 2) its data volume is compressed and coding errors are added to the signal, and 3) pixel values of the decoded image are once again mapped to generate the LDR image.

This kind of scheme has been pointed out in a recent report [7] in international standardization on bit depth scalable coding (BDSC) [8,19]. BDSC can be classified into three categories 1) ratio (or residue after division) based one (R-BDSC) [9,10], 2) low pass based one (L-BDSC) [11], and 3) difference based one (D-BDSC) [7,12-18].

R-BDSC is widely accepted since LDR image is decoded with a conventional lossy coding from its base layer bit stream. The HDR image is also decoded adding one more layer which contains the ratio image of LDR luminance and HDR luminance. On the other hand, L-BDSC and D-BDSC replace the ratio image in the additional layer with low-pass filtered image and difference image, respectively. Especially, D-BDSC is applied to lossy coding of video signals [12-15], and furthermore extended to lossless coding of HDR images [18,7]. Unfortunately, D-BDSC magnifies coding errors in spite of its convenience. This is because it contains inverse mapping in its decoding process.

In this report, we investigate how the errors added in the lossy coding are magnified inside the D-BDSC. As a result of our investigation, it is indicated that the errors in dark pixels of the LDR image are extremely high. More specifically, error gain depending on pixel value is theoretically clarified. Next, we propose to equalize probability density function of the error in the LDR image so that quality of the LDR image can be increased. Finally, we experimentally endorse that the proposed method increases peak signal to noise ratio (PSNR) of the LDR image in the rate distortion curves.

II. PROBLEM SETTING

Fig.1 summarizes the situation we are discussing on. Firstly, extremely high dynamic range of pixel values of an HDR image is normalized to a standard low dynamic range, e.g. 8 [bit], so that a conventionally standard lossy encoder, e.g. JPEG, can be applied for data compression. In in report, the power function

\[ \text{Pow}(x) = x^{1/\gamma} \]  

(1)

is applied as the range normalizer. Secondly, ‘inverse’ of the power function is applied to the normalized dynamic range (NDR) image to generate an approximation of the original HDR image. In the D-BDSC system, this image is utilized to predict the original HDR image, and the prediction error is encoded as the additional layer data. Finally, the HDR approximation is once again tone mapped to generate a visually appropriate LDR image. We use the Hill function:

\[ \text{Hill}(x) = \frac{x^a}{x^a + b^a} \]  

(2)

as an example in report.
In this situation, coding errors added in the lossy coding (encoding and decoding) propagate through the inverse of the power function and the ‘tone mapping’. In brief, the coding errors are magnified by the composite of these functions in the LDR image. As theoretically indicated in the next section, error value in the LDR image varies depending on pixel value. For example, error values tend to be large in dark pixel values, and small in bright pixels.

III. PROPOSED METHOD

A. Introduction of Error Equalization

In this report, we propose to introduce ‘error mapping’ as illustrated in Fig.2 to reduce errors in the LDR image. As explained in the next subsection, the ‘error mapping’ is designed so that probability density function (PDF) or histogram of error values in the LDR image has uniform envelope. It means that error value does not depend on pixel value of the LDR image unlike the existing method.

B. Model for Theoretical Analysis

Next, we investigate how the errors, which are generated in the lossy coding, are magnified through the mappings. Fig.3 illustrates the model for theoretical analysis of the existing method. The HDR image $x_H$ is mapped with the power function $f$ to generate the NDR image $x_N$. After the lossy coding, the decoded NDR image $y_N$ contains coding error $e_N$. It is converted to the approximation of HDR image $y_H$ with the inverse $f^{-1}$. Finally, the LDR image $y_L$ is obtained through the tone mapping $g$ (= the Hill function in this report).

Difference between the existing method and the proposed method is simple. Only the ‘error mapping’ $h$ and its inverse $h^{-1}$ is introduced in our method as illustrated in Fig.4.

C. Analysis on Error Propagation

Fig.5(a) indicates relation between the NDR image $x_N$ and $y_N$. In this example, $y_N$ is generated as

$$y_N = \text{Round}\left\{\frac{x_N}{\text{Qnt}}\right\} \cdot \text{Qnt}$$

and the mapping functions are set to

$$f(x_H) = \frac{x_H^{1/\gamma}}{C_f}, \quad h^{-1}(x_B) = \frac{x_B^{a}}{x_B^{a} + b^a}$$

for

$$\bar{x}_H = \frac{x_H - M_H}{M_X - M_H}, \quad x_H \in [M_H, M_X], \quad C_f = \frac{1}{1 + b^a}.$$
Fig.5(b) indicates relation between pixel value of the NDR image \(x_N\) and the error value \(e_N = y_N - x_N\). As indicated with ‘red’ lines in the figure, error values exist in the interval of \(\pm 8 = \pm \text{Qnt}/2\). On the contrary, as indicated in Fig.5(c) and (d), the error value \(e_L = y_L - x_L\) in the LDR image tend to be large in dark pixel values (around 60) in the existing method. This is because the error \(e_L\) is magnified by differential of the gamma \(f\) and the tone mapping \(g\) as

\[
e_L = \frac{g'}{f'} \cdot e_N. \tag{5}
\]

Proof:

\[
e_L = y_L - x_L = g(f^{-1}(e_N + f(x_L)) - x_L
\]

\[
= \frac{dg(x_H)}{dx_H} \cdot \frac{df^{-1}(x_N)}{dx_N} \cdot e_N
\]

\[
= \frac{dg(x_H)}{dx_H} \cdot \left(\frac{df(x_H)}{dx_H}\right)^{-1} \cdot e_N
\]

Q.E.D.

It was indicated how the coding error in the LDR image is magnified by the power function \(f\) and the tone mapping \(g\) in (4) in the existing method.

D. Error Equalization in the Proposed Method

To increase quality of the LDR image, we equalize PDF of the error value in the LDR image (= not in the NDR image), as follows. Similarly to (5), the error \(e_L\) in Fig.4 is described with the coding error \(e_B\) as

\[
e_L = \frac{g'}{f'} \cdot e_B. \tag{6}
\]

When the PDF of the error \(e_B\) has ‘uniform’ envelope, PDF of the error \(e_L\) is ‘non uniform’. On the contrary, we design the ‘error mapping’ function which makes it ‘uniform’. This is done by setting the ‘error mapping’ function as

\[
h(x) = \int_0^x \frac{g'(t)}{f'(t)} \, dt
\]

\[
h^{-1}(x) = \int_0^x \frac{f'(t)}{g'(t)} \, dt \tag{7}
\]

This is because we set

\[
\frac{g'}{f'} \cdot h = \text{constant} \rightarrow h' = \frac{1}{(h^{-1})'} = \frac{g'}{f'} = \text{constant}. \tag{8}
\]

Effectiveness of this setting is validated with ‘black’ lines in Fig.5. As clearly indicated in Fig.5(d), the error value \(e_L\) of the proposed method has ‘uniform’ envelope in PDF. Unlike the existing method, the error value does not depend on pixel value range of the LDR image. Its effectiveness on HDR images is investigated in the next section.

IV. EXPERIMENTAL RESULTS

In our experiments, HDR images in the ‘Open EXR’ format are tested. An example of the NDR image is indicated in Fig.6(a). The LDR image is indicated in Fig.6(b). Fig.7 indicates the error \(e_N\) in the NDR images. Fig.8(a) indicates the error \(e_L\) in the LDR image of the existing method. It is obviously larger than that of the proposed method in Fig.8(b). It indicates that errors in dark area are larger than bright area in the existing method, as we have theoretically indicated in the previous section.

Next, data compression performance of the systems is investigated as follows. Fig.9 summarizes the rate distortion curves for three kinds of HDR image samples. The horizontal axis indicates the bit rate (= data size per pixel) of the image \(x_B\) in Fig.3 and Fig.4. The vertical axis indicates quality of the LDR image in the peak signal to noise ratio (PSNR). It clearly indicates superiority of the proposed method to the existing method. PSNR is improved by approximately 3 to 4 [dB] by introducing the ‘error mapping’ at around 35 [dB].
V. CONCLUSIONS

In this report, we introduced the ‘error mapping’ to equalize distribution of the error value before data compression of the image with the currently standard encoder. Relation between the problem in the existing method and designing procedure of the ‘error mapping’ were theoretically described. It was confirmed that the error value in the LDR image was equalized, and quality of the LDR image was increased by introducing the ‘error mapping’. We also investigated data compression performance of the system with the rate distortion curves. As a result of experiments, it was observed that the PSNR of the LDR is increased by approximately 3 to 4 [dB] for some HDR image samples.

Since our experiments were limited to a single color component, further investigation should be done for color images in the near future.

REFERENCES