Four-band decomposition module with minimum rounding operations

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A four-band decomposition module for 2D digital signals is proposed. The total amount of the error due to the rounding operation is reduced under the constraint that the module has compatibility with the conventional wavelet transform in JPEG 2000. The number of rounding operations and the standard deviation of the error are reduced by 33.3 and 22.9%, respectively, per one module.

Introduction: The JPEG 2000 international standard implements lossless coding of image signals by the wavelet transform in the form of a lifting structure, including the rounding operations [1, 2]. Lossy coding is also realised by truncating some of the bit planes and frequency bands. The quantisation is prepared as an option for precise bit rate control. When both quantisation and rounding operation are simultaneously adopted, the error due to the rounding operation is not cancelled between encoder and decoder, resulting in degradation of the reconstructed image signal's quality. In general, the total amount of the error is proportional to the number of rounding operations in the wavelet transform. The conventional transform based on a 'separable' 2D filter bank includes 'six' rounding operations, as shown in Fig. 1. On the other hand, the proposed module based on a 'non-separable' 2D filter bank has only 'four' rounding operations, as shown in Fig. 2. This is the minimum number for the case of four-band decomposition. This Letter determines the transfer function of the sub-filters Q_{ij}, where $i,j \in \{1,2,3,4\}$ in Fig. 2, so that the proposed module has exactly the same frequency decomposition characteristics as the conventional JPEG 2000 module in Fig. 1. Improvement by the proposed method in respect of reduction of the error is also confirmed with PSNR for standard image signals.

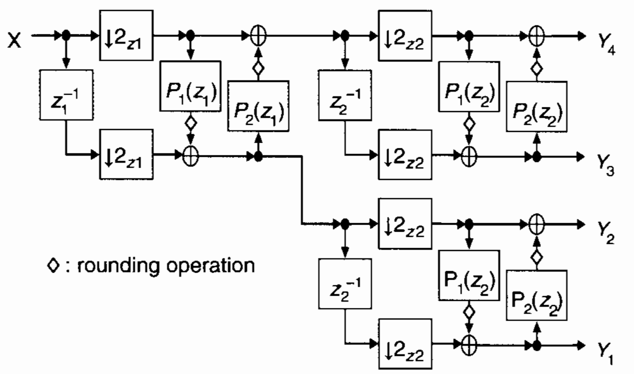

Fig. 1 *Four-band decomposition module based on separable 2D filter bank*

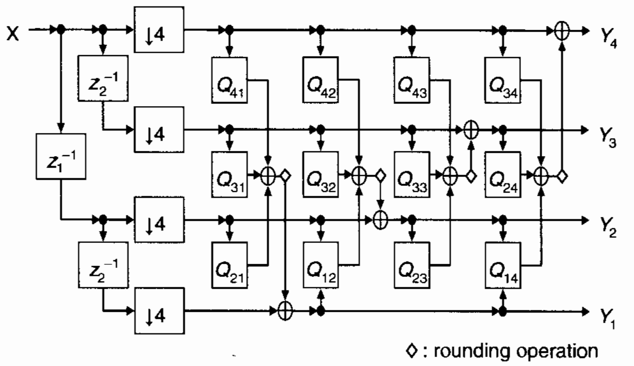

Fig. 2 *Four-band decomposition module based on non-separable 2D filter bank*

Conventional method: Fig. 1 shows the forward wavelet transform of the JPEG 2000. It decomposes an input signal $X(\mathbf{z})$ into four kinds of band signals $Y_b(\mathbf{z})$ where $b \in \{1,2,3,4\}$ and $(\mathbf{z}) = (z_1, z_2)$. It is composed of the sub-filters $P_1(z)$ and $P_2(z)$, which are specified as

$$\begin{bmatrix} P_1(z) \\ P_2(z) \end{bmatrix} = \begin{bmatrix} -1/2 & 0 \\ 0 & 1/4 \end{bmatrix} \begin{bmatrix} 1+z^{-1} \\ 1+z \end{bmatrix}$$

for use of lossless coding and referred to the 5/3 filter for example. These are applied horizontally (or vertically) along with z_1 and vertically (or horizontally) along with z_2. The rounding operation output integer values are close to its input after each of the sub-filters. Denoting the down sampling and matrixes $T(z)$ and $Z(z)$ as

$$\downarrow 2_2(z) = \mathrm{diag}[\downarrow 2(z) \quad \downarrow 2(z)],$$

$$\downarrow 2(z)X(z) = \frac{1}{2}\sum_{p=0}^{1} X(e^{j\pi p} z^{1/2})$$

$$T(z) = \begin{bmatrix} 1 & P_2(z) \\ 0 & 1 \end{bmatrix} \begin{bmatrix} 1 & 0 \\ P_1(z) & 1 \end{bmatrix}, \quad Z(z) = \begin{bmatrix} 1 \\ z^{-1} \end{bmatrix}$$

the band signals are calculated by

$$\begin{aligned} &[Y_4(\mathbf{z}) \quad Y_3(\mathbf{z}) \quad Y_2(\mathbf{z}) \quad Y_1(\mathbf{z})]^T = \mathrm{diag} \\ &[T(z_2) \downarrow 2_2(z_2) Z(z_2) \quad T(z_2) \downarrow 2_2(z_2) Z(z_2)] \\ &T(z_1) \downarrow 2_2(z_1) Z(z_1) X(\mathbf{z}) \end{aligned} \tag{1}$$

Using the properties of the down sampler [3],

$$\begin{aligned} \downarrow 2(z_1) \downarrow 2(z_2) X(\mathbf{z}) &= \frac{1}{4}\sum_{p=0}^{1}\sum_{q=0}^{1} X(e^{j\pi p} z_1^{1/2}, e^{j\pi q} z_2^{1/2}) \\ &\equiv \downarrow 4(\mathbf{z}) X(\mathbf{z}) \\ H(z) \downarrow 2(z) X(z) &= \downarrow 2(z) H(z^2) X(z) \end{aligned}$$

the band signals are represented as

$$\begin{aligned} &[Y_4(\mathbf{z}) \quad Y_3(\mathbf{z}) \quad Y_2(\mathbf{z}) \quad Y_1(\mathbf{z})]^T \\ &= \mathrm{diag}[\downarrow 2_2(z_2) U(z_2) \quad \downarrow 2_2(z_2) U(z_2)] \\ &\quad \downarrow 2_2(z_1) U(z_1) X(\mathbf{z}) \\ &= \downarrow 4_4(\mathbf{z}) \mathrm{diag}[U(z_2) \quad U(z_2)] U(z_1) X(\mathbf{z}) \end{aligned} \tag{2}$$

where

$$U(z) = T(z^2)[1 \quad z^{-1}]^T,$$

$$\downarrow 4_4(\mathbf{z}) = \mathrm{diag}[\downarrow 4(\mathbf{z}) \quad \downarrow 4(\mathbf{z}) \quad \downarrow 4(\mathbf{z}) \quad \downarrow 4(\mathbf{z})]$$

As a result, the band signals are uniquely described by the sub-filters $P_1(z)$ and $P_2(z)$ defined by JPEG 2000 as

$$\begin{aligned} &[Y_4(\mathbf{z}) \quad Y_3(\mathbf{z}) \quad Y_2(\mathbf{z}) \quad Y_1(\mathbf{z})]^T \\ &= \downarrow 4_4(\mathbf{z}) \begin{bmatrix} \{1 + P_1(z_1^2) P_2(z_1^2)\} T(z_2^2) & P_2(z_1) T(z_2^2) \\ P_1(z_1) T(z_2^2) & T(z_2^2) \end{bmatrix} \\ &\quad \mathrm{diag}[Z(z_2) \quad Z(z_2)] Z(z_1) X(\mathbf{z}) \end{aligned} \tag{3}$$

Proposed method: To reduce the rounding errors, we construct the band decomposition module with non-separable 2D sub-filters Q_{ij}, $i,j \in \{1,2,3,4\}$ as shown in Fig. 2. In this case, the number of the rounding operation in one module is reduced from 'six' in Fig. 1 to 'four'. The next problem is how to determine the transfer function of the sub-filters Q_{ij} from the given sub-filters P_1 and P_2 under the constraint that the equation below, which is the transfer function of the module in Fig. 2, is exactly same as (3) of the conventional module:

$$\begin{aligned} \begin{bmatrix} Y_4(\mathbf{z}) \\ Y_3(\mathbf{z}) \\ Y_2(\mathbf{z}) \\ Y_1(\mathbf{z}) \end{bmatrix} &= \begin{bmatrix} 1 & Q_{34} & Q_{24} & Q_{14} \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \\ &\begin{bmatrix} 1 & 0 & 0 & 0 \\ Q_{43} & 1 & Q_{23} & Q_{13} \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ Q_{42} & Q_{32} & 1 & Q_{12} \\ 0 & 0 & 0 & 1 \end{bmatrix} \\ &\begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ Q_{41} & Q_{31} & Q_{21} & 1 \end{bmatrix} \\ &\downarrow 4_4(\mathbf{z}) \mathrm{diag}[Z(z_2) \quad Z(z_2)] Z(z_1) X(\mathbf{z}) \end{aligned} \tag{4}$$

This Letter offers a solution to this problem as shown below:

$$\begin{bmatrix} Q_{41} \\ Q_{31} \\ Q_{21} \end{bmatrix} = \begin{bmatrix} P_1(z_1)P_1(z_2) \\ P_1(z_1) \\ P_1(z_2) \end{bmatrix}, \begin{bmatrix} Q_{42} \\ Q_{32} \\ Q_{12} \end{bmatrix} = \begin{bmatrix} P_1(z_1) \\ 0 \\ P_2(z_2) \end{bmatrix},$$
$$\begin{bmatrix} Q_{43} \\ Q_{23} \\ Q_{13} \end{bmatrix} = \begin{bmatrix} P_1(z_2) \\ 0 \\ P_2(z_1) \end{bmatrix}, \begin{bmatrix} Q_{34} \\ Q_{23} \\ Q_{14} \end{bmatrix} = \begin{bmatrix} P_2(z_2) \\ P_2(z_1) \\ -P_2(z_1)P_2(z_2) \end{bmatrix} \quad (5)$$

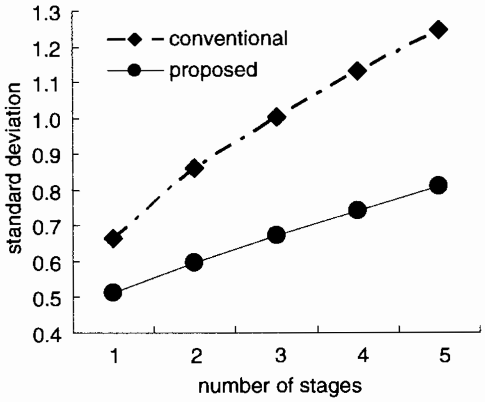

Fig. 3 *Standard deviation of rounding error*

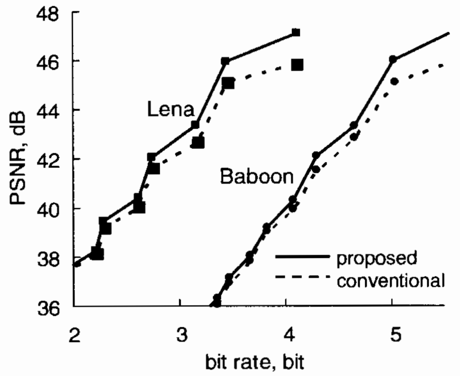

Fig. 4 *PSNR of reconstructed images*

Results: It is already confirmed that the number of rounding operations is reduced by 33.3%. Fig. 3 shows the standard deviation of the rounding error in a reconstructed image signal. For the case of four-band decomposition, it is reduced from 0.66 to 0.51. The reduction is 22.9%. When the module is cascaded to the lowest band three, four and five times, the reduction rate is 33.0, 34.3 and 35.0%, respectively. Fig. 4 shows the PSNR (peak-signal-to-noise ratio) against bit rate in lossy coding with three-stage octave decomposition of standard image signals 'Lena' and 'Baboon', for example. The PSNR is improved from 45.8 to 47.1 dB at the bit rate of 4.1 bpp for 'Lena'. Improvement for 'Baboon' at 5.6 bpp is 1.3 dB. It is confirmed that the proposed module can reduce errors owing to the rounding operation maintaining compatibility with the conventional JPEG 2000.

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References

1 Taubman, D.S., and Marcellin, M.W.: 'JPEG 2000-image compression fundamentals, standards and practice' (Kluwer Academic, 2002)
2 ISO/IEC FDIS 15444-1: 'Information technology—JPEG 2000 image coding system', ISO/IEC JTC1/SC29/WG1 (January 2001)
3 Vaidyanathan, P.P.: 'Multirate systems and filter banks' (Prentice Hall Signal Processing Series, 1993)