A NEW CLASS OF LIFTING WAVELET TRANSFORM FOR GUARANTEEING LOSSLESSNESS OF SPECIFIC SIGNALS

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ABSTRACT

This paper proposes a new class of lifting wavelet transform which can guarantee losslessness of specific signals, e.g. white balance. The 5/3 wavelet transform composed of two lifting steps can reconstruct an input signal without any loss and has been utilized for lossless coding. The 9/7 wavelet contains two more lifting steps and two scaling pairs for effective lossy coding. However the losslessness is not guaranteed due to rounding of signal values and scaling coefficient values. This paper analyzes condition on word length (WL) and bit depth (BD) for the losslessness and proposes a new class of wavelet transform with "DC lossless" property which is a kind of specific losslessness. This can be utilized as a standard condition for algorithms or LSI processors to guarantee no error from the wavelet transform for white balance signals.

***Index Terms*—**wavelet , lossless, specific, coding

1. INTRODUCTION

Over the past few years, a considerable number of studies have been made on the lifting wavelet transforms [1,2]. The lifting structured wavelet, such as the 5/3 wavelet in JPEG 2000 (JP2K), can reconstruct an input signal without any loss [3]. Therefore it has been utilized for lossless coding of digital images. A transform of the 5/3 wavelet is composed of two lifting steps and therefore its frequency characteristics are poor. It is necessary to introduce more lifting steps and scaling to increase coding gain.

The 9/7 wavelet for lossy coding in JP2K has four lifting steps and two scaling pairs. However the losslessness is not guaranteed due to two reasons. One is truncation of scaling coefficient values into finite word length (WL) [4,5]. The other is rounding of signal values into finite bit depth (BD). These are necessary for digital computation and their minimum requirement should be investigated.

When input signal values are limited to integers, truncating final output values from the inverse wavelet into integers, it is possible to have no loss by assigning enough BD to band signals. However, it degrades coding efficiency.

Beside the coding efficiency, it is important to discuss losslessness of the signals when numerical precision is necessary to be guaranteed. For example, compatibility between an encoder and a decoder is defined by range of errors [6,7].

In this paper, firstly we derive WL condition (WL-C) on scaling coefficients and BD condition (BD-C) on signals in the wavelet for guaranteeing the losslessness. When a lifting wavelet satisfies both of the conditions, namely both of WL and BD are greater than the minimum values determined by the conditions, it becomes lossless for its input signal.

Secondly, we propose a new class of lifting wavelet transform which guarantees losslessness for specific signals, not for any signals but for DC signals as illustrated in figure 1. This "DC lossless" property, an example of specific losslessness, can be utilized as a standard condition for algorithms or LSI processors to guarantee no error from the wavelet transform for white balance signals.

Our investigation also includes a new scaling-lifting (SL) type wavelet comparing to a conventional lifting-scaling (LS) type. In general, the lifting pair has a property that any rounding errors cancel between forward and backward transforms. However, when some errors occur from the scaling pair amid the lifting pair, it can not be lossless. On the contrary, we changed the order of lifting and scaling so that there is no error in the lifting pair.

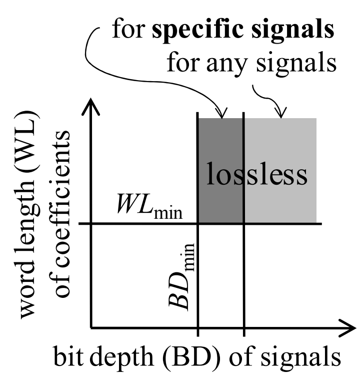


Figure 1 Purpose of the research.

2. ANALYSIS ON LOSSLESS CONDITIONS

Defining the word length $WL > 0$ of a truncated coefficient h for a given $h^* \in \mathbf{R}$ by

$$h = \sum_{w=0}^{WL} h_w 2^{-w}, \quad 0 < h < 2, \quad h_w \in \{0,1\} \tag{1}$$

and the bit depth BD of a signal value x by

$$x = \sum_{b=-BF}^{BI-1} x_b 2^b, \quad 0 \le x < 2^{BI}, \quad x_b \in \{0,1\} \tag{2}$$

where $BD=BI+BF>0$, we firstly derive conditions on WL and BD for guaranteeing losslessness of input signals. It is assumed that one bit is assigned to sign of h and x beside WL and BD respectively.

2.1. Word Length Condition (WL-C) on a Scaling

Figure 2 (a) illustrates a scaling of an integer input value x_i with $BF_i=0$ by a rational number coefficient value h. This is a mapping f_h of a set X_i to a set X_o defined by

$$f_h : X_i \to X_o \tag{3}$$

where

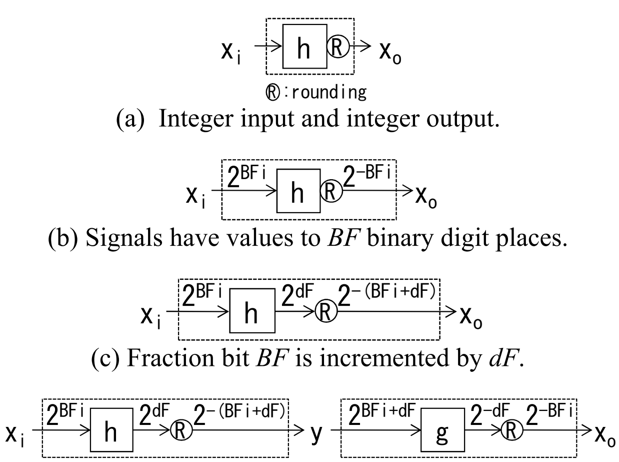
$$\begin{aligned} X_i &= \{x_i \mid 1,2,\cdots,2^{BIi}-1\} \in \mathbf{N}, \\ X_o &= \{x_o \mid f_h(x_i)\} \in \mathbf{N} \end{aligned} \tag{4}$$

$$f_h(x_i) = R[hx_i] = (hx_i + 2^{-1}) - (hx_i + 2^{-1}) \bmod 1 . \tag{5}$$

In this report, we define the WL condition (WL-C) by

$$f_h(x_i) = f_{h^*}(x_i), \quad \forall x_i \in X_i \tag{6}$$

and determine the minimum WL which satisfies this condition in **3**.



(a) Integer input and integer output.

(b) Signals have values to BF binary digit places.

(c) Fraction bit BF is incremented by dF.

(d) A scaling pair.

Figure 2 Scaling procedures of a signal value x_i.

This can be expanded to a rational number input x_i with $BF>0$ as illustrated in figure 2(b). Furthermore, it is also possible to introduce increment of the fraction bit by dF as indicated in figure 2(c). Integer bit BI_o and fraction bit BF_o of output value x_o are given by

$$(BI_o, BF_o) = (\lceil BI_i + \log_2 h \rceil , BF_i + dF_i) . \tag{7}$$

In this case, the WL-C becomes

$$f_h\left(2^{dF} x'_i\right) = f_{h^*}(2^{dF} x'_i), \quad \forall x'_i \in X'_i . \tag{8}$$

where

$$X'_i = \{x'_i \mid x'_i = 2^{BFi} x_i = 1,2,\cdots,2^{BIi+BFi}-1\} \in \mathbf{N} . \tag{9}$$

Substituting equation (5), this condition is expressed by

$$R[(h^* - \Delta h) 2^{dF} x'_i] = R[h^* 2^{dF} x'_i], \quad \forall x'_i \in X'_i \tag{10}$$

where

$$\Delta h = \sum_{w=WL+1}^{\infty} h_w 2^{-w} > 0, \quad h^* = \sum_{w=0}^{\infty} h_w 2^{-w} . \tag{11}$$

This gives the upper bound as

$$\begin{aligned} \Delta h &< \min \frac{(h^* 2^{dF} x'_i + 2^{-1}) \bmod 1}{2^{dF} x'_i}, \quad \forall x'_i \in X'_i \\ &< 2^{-(BIi+BFi+dF)} \end{aligned} \tag{12}$$

As a result, substituting

$$\max \Delta h = 2^{-WL} \tag{13}$$

into equation (12), the WL-C for a scaling becomes

$$WL > BI_i + BF_i + dF . \tag{14}$$

It is concluded that the minimum word length WL_{min} is determined by integer bit BI_i and fraction bit BF_i of an input signal value x_i and also fraction bit increment dF.

2.2. WL-C on a Scaling Pair

The wavelet transform includes a scaling pair (h, g) illustrated in figure 2(d) where $g=h^{-1}$. This is also a mapping defined by

$$x_o = 2^{-BFi} R[2^{-dF} g R[2^{dF} h 2^{BFi} x_i]] . \tag{15}$$

This becomes lossless, namely $x_o=x_i$, under sufficient bit depth described in the next subsection. In this case, similarly to equation (10), the WL-C on a coefficient h and g in the scaling pair becomes

$$x'_i = R[2^{-dF} g^* R[2^{dF} (h^* - \Delta h) x'_i]], \quad \forall x'_i \in X'_i \tag{16}$$

$$x'_i = R[2^{-dF} (g^* - \Delta g) R[2^{dF} h^* x'_i]], \quad \forall x'_i \in X'_i \tag{17}$$

respectively. Both of them lead the same upper bound expressed in equation (14).

2.3. Bit Depth Condition (BD-C) on a Scaling Pair

In general, the mapping f_h in figure 2(a) for $2^{-1}<h<2^{1}$ has a property that

(a) f_h is bijective $\Leftrightarrow$ h=1,
(b) f_h is injective $\Leftrightarrow$ h>1,
(c) f_h is surjective $\Leftrightarrow$ h<1.

The mapping from x_i to x_o in the scaling pair in figure 2(d) is a composite function of f_h and f_g defined by

$$f_g \circ f_h : X_i \rightarrow Y \rightarrow X_o . \tag{18}$$

For losslessness, this mapping must be bijective and this is satisfied if and only if "f_h is injective and f_g is surjective". Since $g=h^{-1}$, this condition reduces to "f_h is injective". Expanding this theory to the scaling pair in figure 2(d), where its mapping is expressed by

$$f_{g2^{-dF}} \circ f_{h2^{dF}} : X'_i \rightarrow Y \rightarrow X'_o , \tag{19}$$

the BD-C on the scaling pair (h, g) becomes

(d) $f_{h2^{dF}}$ is injective $\Leftrightarrow$ $h2^{dF}>1$.

It is concluded that the BD-C is given by

$$dF > -\log_2 h . \tag{20}$$

For example, in the scaling pair (K^{-1}, K) and (K, K^{-1}) in figure 3 and 4, dF must satisfy

$$\begin{cases} h = K^{-1} = 0.8129 & \Rightarrow \quad dF = \lceil +0.2989 \rceil = 1 \\ h = K = 1.2302 & \Rightarrow \quad dF = \lceil -0.2989 \rceil = 0 \end{cases} . \tag{21}$$

2.4. Error of the Scaling Pair

In case of the BD-C is not satisfied (e.g. h<1, dF=0), defining the signal error from the scaling pair by

$$\Delta x = x_o - x_i \ , \tag{22}$$

the error has an interesting property for $2^{-1}<h<2^{0}$ that its probability density function $p(e)$ satisfies

$$\begin{cases} p(\Delta x = e) \neq 0 & e \in \{-2^{-BFi}, 0, 2^{-BFi}\} \\ p(\Delta x = e) = 0 & e \notin \{-2^{-BFi}, 0, 2^{-BFi}\} \end{cases} \tag{23}$$

and therefore,

$$\|\Delta x\|^{\infty} = \max|\Delta x| = 2^{-BFi} \ . \tag{24}$$

The scaling pair (K^{-1}, K) in the low band channel in figure 3 and 4 have (BF_i, dF)= (U, V) and (BF_i, dF)= (0,U) for V=0 respectively.

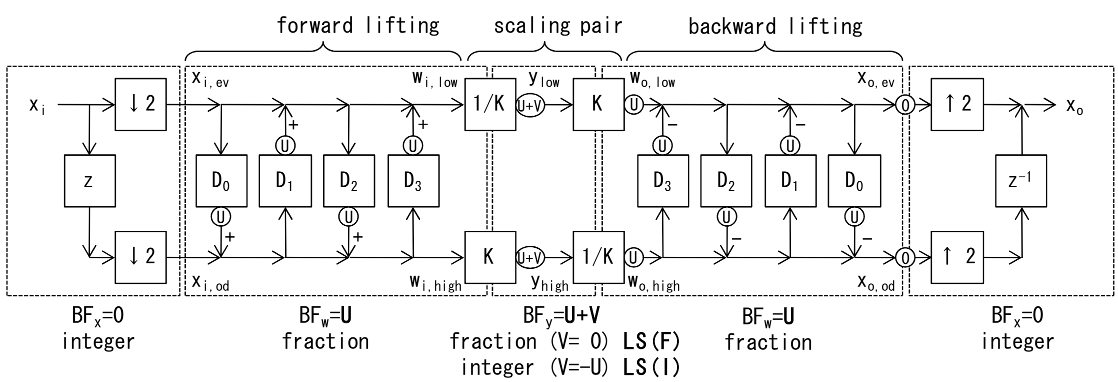


Figure 3 Lifting-Scaling (LS) type wavelet transform (conventional).

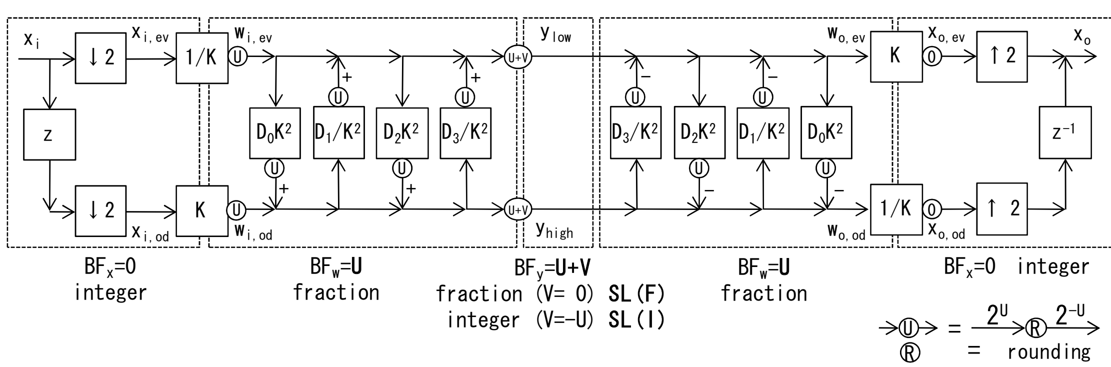


Figure 4 Scaling-Lifting (SL) type wavelet transform (Newly introduced).

3. A NEW CLASS OF WAVELET

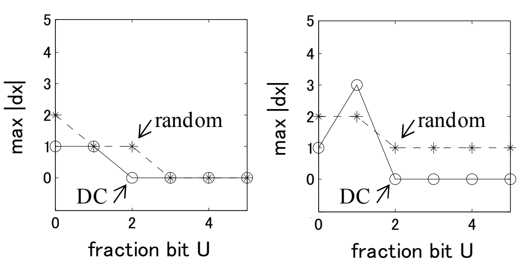
Finally, utilizing WL-C and BD-C investigated in the previous section, we propose a new type of wavelet transform which guarantees losslessness of specific signals.

3.1. LS(F) Wavelet

Under long enough WL of all the multiplier coefficients, the minimum bit depth (min. BD) of a signal is experimentally investigated. The conventional wavelet LS(F) with V=0 in figure 3, Lifting-Scaling with Fraction band signal, requires 3 and 2 [bit] to guarantee losslessness for random signals and DC signals respectively as illustrated in figure 5(a). BI_i is set to 2^8. Under the min. BD, a coefficient K in the backward transform is truncated to determine the minimum word length (min. WL). Results are illustrated in figure 6. The min. BD and min. WL are summarized in ex.1 and ex.2 of table 1. Under long enough BD in ex.1, the lifting pair becomes lossless. It is not necessary to care WL of coefficients in the lifting steps D_0-D_3 (indicated as 0 [bit] in the table).

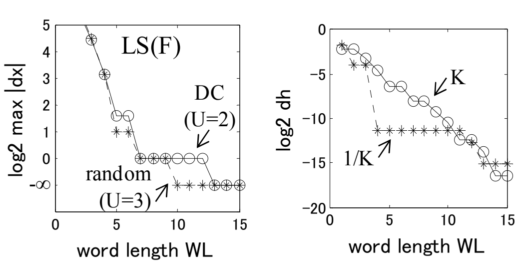
3.2. LS(I) Wavelet

Similarly, the LS(I) with V=-U in figure 3, Lifting-Scaling with Integer band signal, requires 2 [bit] for DC signals. It is advantageous for multi-stage decomposition to output integer band signals. However it does not become lossless for any signals as indicated in figure 5(b). Maximum value of the loss is two and it is negligible in case of lossy coding. WL are summarized in ex.3 of table 1. There is no requirement to the multipliers in the high band channel.



(a) LS(F) V=0 (b) LS(I) V=-U

Figure 5 Fraction bit depth and signal errors.



(a) signal error of LS(F) (b) coefficient error

Figure 6 Word length and errors.

Table 1 Minimum BD and WL for lossless

ex.	method	input	min. BD		min. WL							
			BF_w (U)	BF_y (U+V)	forward		backward		lifting			
					K^{-1}	K	K	K^{-1}	D_0	D_1	D_2	D_3
1	LS(F)	random	3		4	10	10	4	0			
2		DC	2		4	0	13	0	7	12	14	2
3	LS(I)	DC	2	0	4		13		7	12	14	0
4	SL(F)	random	1		4	9	10	4	0			
5		DC	1		4	10	9	4				
6	SL(I)	DC	3	0	4	10	10	4	11	14	7	0

3.3. SL(F) Wavelet and SL(I) Wavelet

The newly introduced SL in figure 4, Scaling-Lifting, are also investigated. Since the lifting pair always becomes lossless in SL(F), WL can be zero for the lifting steps. In case of SL(I), it is necessary that the gain to a DC input of the forward lifting part is equal to K so that the low band signal becomes exactly same as the DC input (DC pass condition). Results are summarized in ex.4, 5, 6 of the table.

4. CONCLUSIONS

We have theoretically analyzed the WL-C and the BD-C for guaranteeing lossless of input signals. The minimum BD and WL are experimentally investigated especially for a set of DC signals, a kind of specific signal. It is our proposal to design a new class of wavelet which satisfies the specific losslessness: "DC lossless" in this report. This property can be a standard condition for wavelet algorithms or processors to guarantee losslessness of a white balance. Our analysis is expanded to multi-stage decomposition in reference [8].

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