# ESTIMATION OF ALIASING ERROR IN LAYERED CODING SYSTEM

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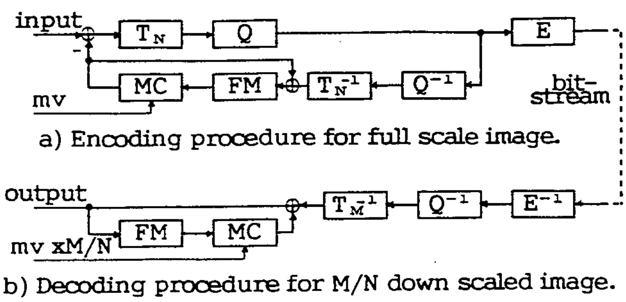
## ABSTRACT

This paper theoretically investigates data distortion caused by resolution conversion in DCT (discrete cosine transform) based layered coding system. The system is attractive because it can not only compress large amounts of video data but also convert the scale of reconstructed pictures. However only a few attempts have so far been made at degree of aliasing error. In this paper we treat the DCT system as a filter bank so that we can evaluate the system in frequency domain. As a result of our study the system is theoretically evaluated from multirate stand point of view and the DCT system is compared with other layered coding systems, e.g. subband coding and Gaussian / Laplacian pyramid, by means of one objective measure.




$T_N$; *N point DCT*    *FM; frame memory*
*Q ; quantization*    *MC; motion compensation*
*E ; entropy coding*    *mv; motion vector*

Fig.1 The DCT system for digital video coding. Scale of input data is converted by combination of N point DCT and M point IDCT.

## 1. INTRODUCTION

Over the past few years a considerable number of studies have been made on Discrete Cosine Transform (DCT) coding to compress large amounts of digital video data [1][7]. Considering large variety of applications compatibility is also required [8][9]. Therefore DCT based layered coding system, which can not only compress data but also convert its scale, has attracted a great deal of attention [2][3].

Our concern is to examine degree of wave-form distortion (aliasing error) caused by converting scale. J.M.Adamt et al. concluded that absolutely no perceptible distortion could be observed on 2:1 scaled-down images [4]. K.N.Ngan derived a relation between DCT and Discrete Fourie Transform (DFT) as a theoretical approach to examine the error [5]. However, both of them used subjective evaluation. In this paper, we analyze the distortion and lead an objective measure to theoretically evaluate degree of the error and compare some systems.

## 2. OUTPUT SIGNAL OF THE SYSTEM

Figure 1 illustrates the DCT layered coding system. Encoder of the system compresses data with NxN DCT. While MxM Inverse DCT (IDCT) is applied to low frequency of DCT coefficients in decoder (M<N). Here we treat this DCT based scaling system as a filter bank indicated in figure 2.

The filter bank decomposes input signal $x(n)$ into N band signals with N tap analysis filters $H_i$. After that, only M band signals are synthesized with M tap synthesis filters $G_i$ and are added together to get output signal $\bar{x}(n)$. It must be noticed that $\bar{x}(n)$ contains aliasing error, namely the difference between $x(n)$ and $\bar{x}(n)$ leads energy of the error. We derive output signal $\bar{x}(n)$ of the system in this section and define a subjective measure as energy of the error in the next section.

Filter coefficients of the bank are expressed by DCT and IDCT coefficients by

$$h_i(j) = e_{i,N-j-1} \quad \text{(analysis)}$$
$$g_i(j) = e'_{i,j} \quad \text{(synthesis)} \qquad (1)$$



Vol. V, pp. 349-352

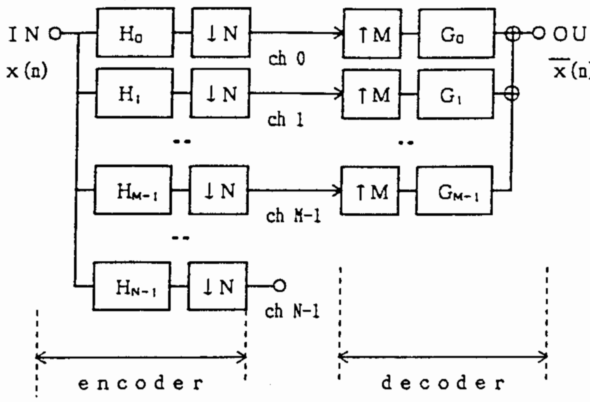



Fig.2 Filter bank which is equivalent to the DCT system in respect of scaling.

where

$e_{i,j} = \sqrt{2/N}\, d(i) \cos[i(2j+1)\pi/2N]$ (DCT)

$e'_{i,j} = \alpha \sqrt{2/M}\, d(i) \cos[i(2j+1)\pi/2M]$ (IDCT)

$d(0)=1/\sqrt{2}$, $d(i)=1$ $(i \neq 0)$, $\alpha$ is a gain factor. Figure 3 indicates frequency characteristics of the filters, which are linear phase, of the DCT system.

Defining z-transform of a signal x(n) as

$$X(z)=\sum_{k=0}^{N-1} x(n)z^{-n} \quad (2)$$

the relation between input and output of the system is given in z-transform domain as follows.

$$\bar{X}(z) = (1/N)z^{s} \sum_{i=0}^{M-1} |G_i(z)| \cdot \sum_{k=0}^{N-1} |H_i(z^{M/N} \cdot W^k)| \cdot X(z^{M/N} \cdot W^k) \cdot W^{kp} \quad (3)$$

where

$$W= e^{-j2\pi/N}$$

$$s = (N-1)M/2N - (M-1)/2$$

$H_i(z)$ and $G_i(z)$ are z-transform of $h_i$ and $g_i$ respectively. $|Hi(z)|$ denotes real part of Hi(z). Aparting k=0 part from others and replacing z by $e^{jw}$ eq.(3) becomes

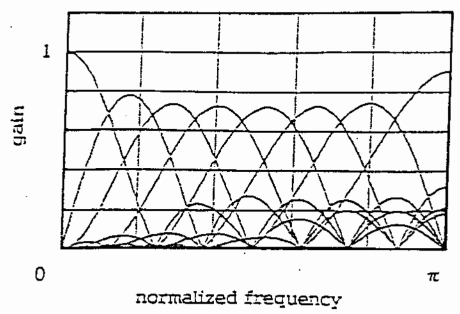
$$\bar{X}(e^{jw})= e^{jws} \{A(e^{jw}) \cdot X(e^{jwM/N})+B(e^{jw})\} \quad (4)$$

where

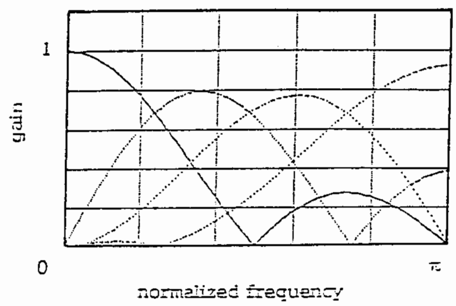
$$A(e^{jw})=\frac{1}{N}\sum_{i=0}^{M-1} |G_i(e^{jw})| \cdot |H_i(e^{jwM/N})| \quad (5)$$

$$B(e^{jw})=\frac{1}{N}\sum_{i=0}^{M-1} |G_i(e^{jw})| \sum_{k=1}^{N-1} |H_i(u)| \cdot X(u)e^{2\pi pkj/N} \quad (6)$$

$$u = e^{j(wM+2\pi k)/N}$$




(a) Analysis filters $H_i$ of the DCT system (N=8).




(b) Synthesis filters $G_j$ of the DCT system (M=4).

Fig.3 Magnitude response of the filters $H_i$ and $G_j$ of the filter bank where $i \in [0,N-1]$ and $j \in [0,M-1]$.

Eq.(5) and eq.(6) are essentially equivalent to all pass part and aliasing part in figure 4 respectively for the DCT system. For other systems, e.g. subband and Gaussian / Laplacian pyramid, Eq.(5) and (6) where N=2 and M=1 are exactly the same as figure 4. Figure 5-7 illustrate each part for the methods where $X(e^{jw})=1$ for all w. They indicate that the DCT system works as a fine low pass filter when it is compared with others.

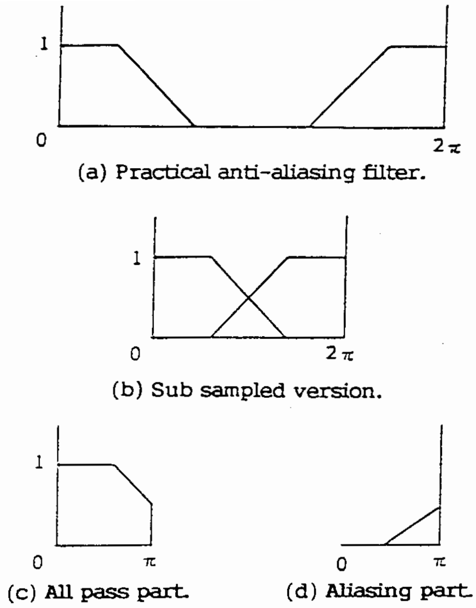
### 3. MEASURE FOR ALIASING ERROR

We define subjective measures for evaluation of aliasing error as energy of difference between input and output of the system by

$$E_{power}= N/(M\pi) \int_0^{\pi M/N} [X(e^{jwM/N})-\bar{X}(e^{jwM/N})]^2 d\omega$$

$$E_{SNR} = -10 \log_{10} E_{power} \quad [\mathrm{dB}] \quad (7)$$

These value indicate total energy of the distortion in either frequency domain or spatial domain (Parseval's theorem [10]).

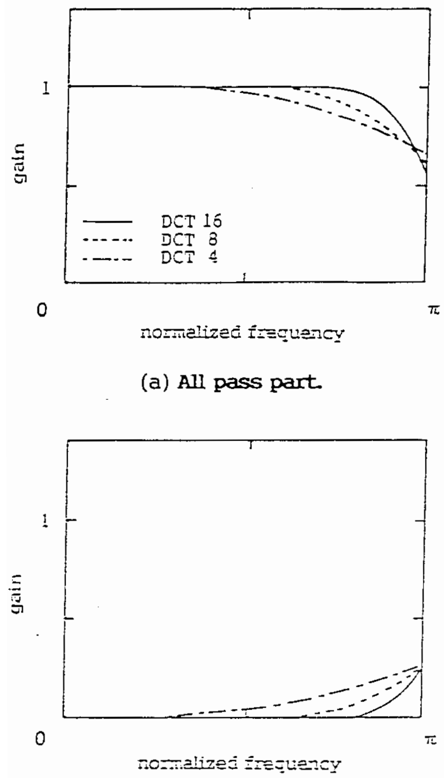



(a) Practical anti-aliasing filter.
(b) Sub sampled version.
(c) All pass part. (d) Aliasing part.

**Fig.4** 2:1 scaling with a low pass filter.

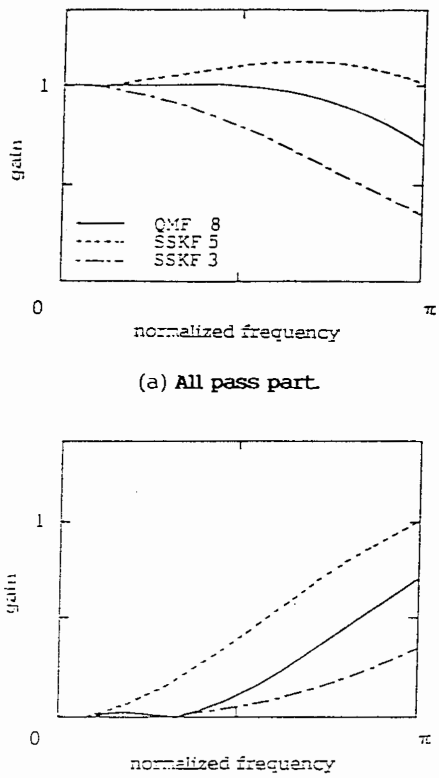
### 4. SIMULATION

Figure 8 indicates the error of 8:M (M<8) scaling in the DCT system. We can confirm that degree of total error decreases as M increases and the error disappears when M=8. Figure 9 compares the DCT system with other layered coding systems. All of methods are better than no-filtering (1 tap) at least. We may say that the DCT system is nearly equal to subband system (QMF) in respect of aliasing error. Figure 10 shows that total error decreases as N increases for case of 2:1 scaling in general. We evaluated error with classical SNR. However we should pay attention to other artifacts, e.g. ringing, which can not be expressed by means of SNR.



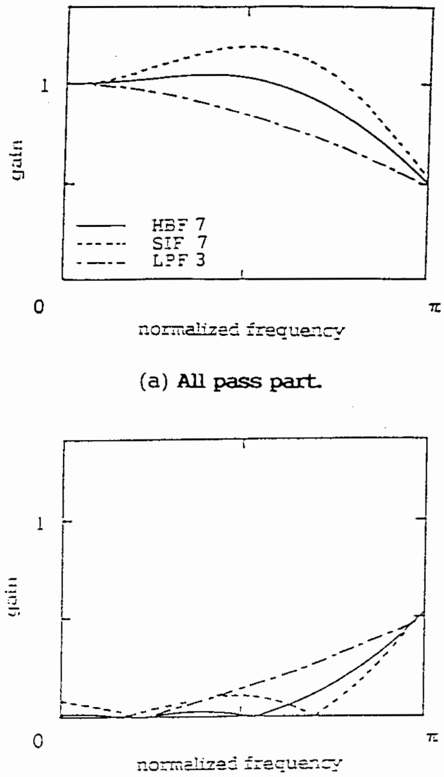

(a) All pass part.
(b) Aliasing part.

**Fig.5** DCT system. (N:M=2:1)




(a) All pass part.
(b) Aliasing part.

**Fig.6** Subband coding with QMF[11] and SSKF[6]




(a) All pass part.
(b) Aliasing part.

**Fig.7** Gaussian pyramid with SIF[7], half band filter(HBF) and low pass filter(LPF).

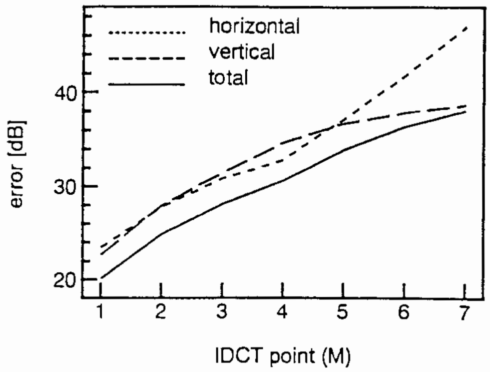



Fig.8 8:M scaling error of the DCT system.

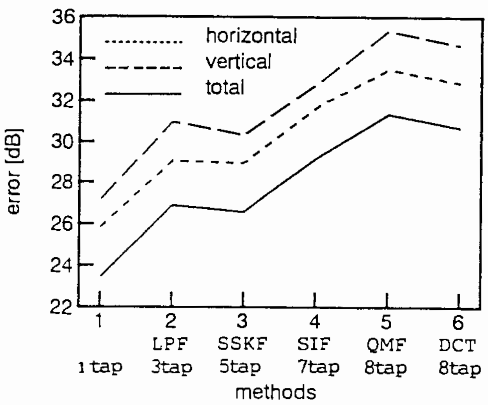



Fig.9 2:1 scaling error of the systems.

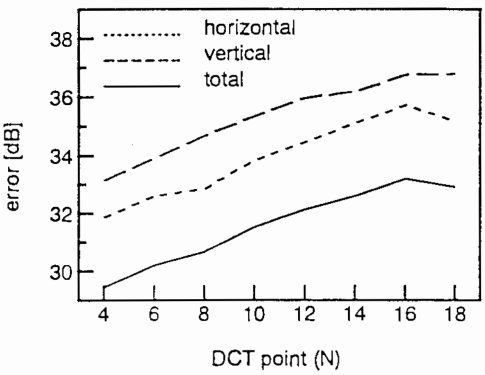



Fig.10 2:1 scaling error of the DCT system.

* 256x256 pixel at the center of interlaced standard picture "mobile and calender" (1st frame) is used in Fig.8-10.

## 5. CONCLUSION

In this paper, we led an objective measure to theoretically evaluate degree of aliasing error and compared the DCT based resolution conversion method to other methods. The measure was driven from treating the DCT system as a filter bank and calculating energy of difference between input and output signal of the system in frequency domain.

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