title

A New Structure of Integer DCT Least Sensitive to Finite Word Length Expression of Multipliers

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Integer DCTs

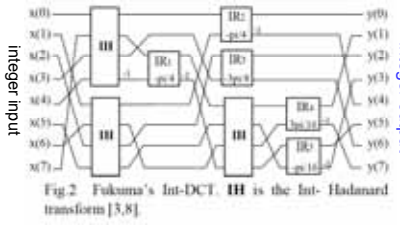


Fig.2 Fukuma's Int-DCT. III is the Int- Hadanard transform [3,8].

☆ Suitable for
- Lossless Coding
- Integer Memory
- Compatible to the normal DCT

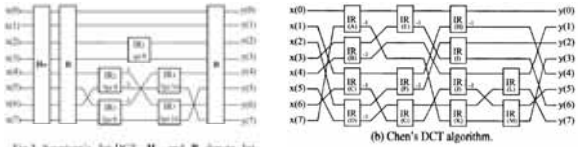
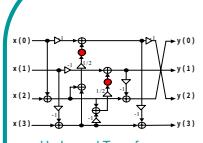


Fig.3 Soontorn's Int-DCT. H, and R denote Int-Hadanard transform and permutation respectively [4].

(b) Chen's DCT algorithm.

Which one is the best ?



$$T=\begin{pmatrix}\cos\theta & -\sin\theta\\ \sin\theta & \cos\theta\end{pmatrix}$$

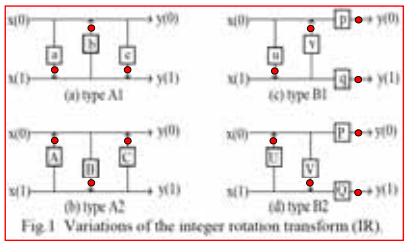
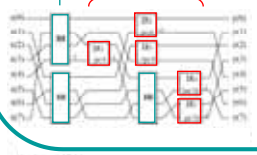


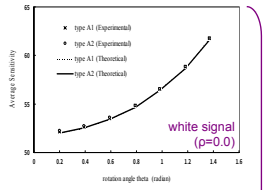
Fig.1 Variations of the integer rotation transform (IR).

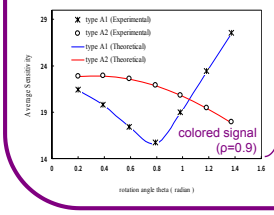
"●" denotes a "rounding" operation.



☆ Four candidates for "IR" transform.
- Which is the best ?
- in respect of "sensitivity"
- for finite word length expression

Considering "Color-ness"...





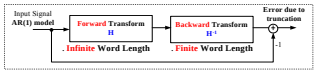
● Definition of the "sensitivity"

$$S_h = \frac{\Delta \sigma_{err}}{\Delta h}$$

where

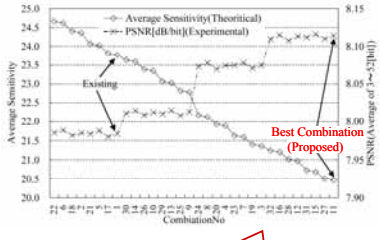
$$\Delta h = \hat{h} - h,$$

$$h \in \{a,b,c,A,B,C,p,q,u,v,P,Q,U,V\}$$



☆ No difference for white signal.
- However, it is different for
- colored signal such as "image"

Conclusion



★Average Sensitivity

$$\bar{S} = \prod_k \sqrt[k]{S_k}$$

★ PSNR after optimum word length allocation

$$PSNR = 10\log_{10}\frac{255^2}{\Delta\sigma^2_{total}}$$

$$W_k = \bar{W} + \log_2\frac{S_k}{\bar{S}}$$

☆ The best combination of the four candidates was found.
- better than conventional by 1.8 dB
- for colored signal such as "image"

Table 1 Results for Fukuma's Int-DCT.

No.	IR3	IR4	IR1	IR2	IR5	Average Sensitivity	PSNR	Remark
12	A1	A2	A1	A1	A2	4.78	**36.55**	Best
8	A1	A1	A1	A1	A1	5.51	**35.77**	Existing
23	A2	A1	A2	A1	A2	5.59	47.05	Worst

Previous Works

● Various Integer DCTs have been proposed by

[4] Y.J.Chen, Soontorn Oraintara, T.Nguyen, "Integer Discrete Cosine Transform (Int DCT)", International Conference on Information, Communications and Signal Processing, Dec. 1999.
[5] W.Philips, "The lossless DCT for combined lossy / lossless image coding", IEEE ICIP 98, vol.3, pp.871-875, Oct 1998.
[6] S.Chokchaitam, M.Iwahashi, S.Jitapanakul, "A New Lossless-DCT for Unified Lossless / Lossy Image Coding", IEEE MWS-CAS 2004, July 2004.

● Optimum word length has been assigned to each of multipliers in the IR by

[8] Iwahashi, O.Nishida, S.Chokchaitam, N.Kanbayashi, "Optimum Word Length Allocation for Multipliers of Integer DCT", IEEE ISCAS 2003, vol.2, pp.400-403, May 2003.

● Robustness of the assigned word length has been theoretically evaluated by

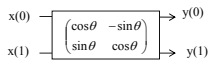
[9] M.Iwahashi, K.Nakagawa, S.Chokchaitam, Y.Tonomura, "Theoretical Analysis on Optimum Word Length Assignment for Integer DCT", IEEE ICIP '04, WA-L4, pp.2507-2510, (2004.10)

● Best combination of the four kinds of "IR" transforms has not been discussed.

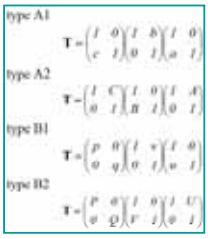
This time, we have done!

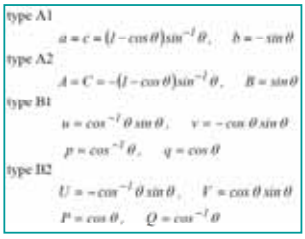
Integer Rotation Transform (IR)

- Rotation Transform



- Integer Rotation Transform (IR)





Theoretical Analysis on the "Sensitivity"

$$S^2_h = \frac{\Delta\sigma_{err}}{\Delta h} = \frac{\sigma^2_x}{2} trace\left[\frac{\partial \mathbf{T}^T}{\partial h}\mathbf{T}\cdot R_{xx}(\rho)\cdot \mathbf{T}^T\frac{\partial \mathbf{T}}{\partial h}\right]$$

- σ_x^2 : variance of input signal x
- $R_{xx}(\rho)=\begin{bmatrix}1 & \rho\\ \rho & 1\end{bmatrix}$: covariance matrix of x
- ρ : correlation coefficient of x

Type A1

$$S^2_a = \frac{\sigma^2_x}{2}$$
$$S^2_b = \frac{\sigma^2_x}{2}(1+\rho\sin\theta)\cos^{-4}\frac{\theta}{2}$$
$$S^2_c = \frac{\sigma^2_x}{2}(1-\rho\sin 2\theta)$$

Type A2

$$S^2_A = \frac{\sigma^2_x}{2}$$
$$S^2_B = \frac{\sigma^2_x}{2}(1-\rho\sin\theta)\cos^{-4}\frac{\theta}{2}$$
$$S^2_C = \frac{\sigma^2_x}{2}(1+\rho\sin 2\theta)$$

Type B1

$$S^2_u = \frac{\sigma^2_x}{2}$$
$$S^2_v = \frac{\sigma^2_x}{2}(1+\rho\sin 2\theta)\cos^{-4}\theta$$
$$S^2_p = \frac{\sigma^2_x}{2}(1-\rho\sin 2\theta)\cos^{-2}\theta$$
$$S^2_q = \frac{\sigma^2_x}{2}(1+\rho\sin 2\theta)\cos^{-2}\theta$$

Type B2

$$S^2_U = \frac{\sigma^2_x}{2}$$
$$S^2_V = \frac{\sigma^2_x}{2}(1-\rho\sin 2\theta)\cos^{-4}\theta$$
$$S^2_P = \frac{\sigma^2_x}{2}(1-\rho\sin 2\theta)\cos^{-2}\theta$$
$$S^2_Q = \frac{\sigma^2_x}{2}(1+\rho\sin 2\theta)\cos^{2}\theta$$

Lossless Coding Performance

Total entropy rate [bit/pixel]
Encoder is Fukuma's Integer DCT

Not good, so excluded.

Image Name	Integer Rotation Transform Type		
	A1 or A2	B1	B2
couple	4.5857	4.6774	4.9010
aerial	6.3983	6.4507	6.8297
girl	4.7076	4.8677	5.0754
barbara	5.3476	5.5229	5.7245
lena	5.3857	5.5692	5.7228
house	5.8692	5.9695	6.1195
church	6.6375	6.7088	6.9796
boat	5.1296	5.2129	5.4310
airplane	5.2122	5.3167	5.5151
Average	**5.4748**	**5.5884**	**5.8110**

Average sensitivity of the combinations

Fukuma's Integer DCT

Best

N	IR5	IR4	IR3	IR2	IR1	Average Sensitivity
15	A1	A2	A2	A2	A1	4.67
11	A1	A2	A1	A2	A1	4.69
16	A1	A2	A2	A2	A2	4.77
12	A1	A2	A1	A2	A2	4.78
31	A2	A2	A2	A2	A1	4.79
27	A2	A2	A1	A2	A1	4.8
32	A2	A2	A2	A2	A2	4.89
28	A2	A2	A1	A2	A2	4.9
7	A1	A1	A2	A2	A1	4.96
3	A1	A1	A1	A2	A1	4.98
8	A1	A1	A2	A2	A2	5.07
4	A1	A1	A1	A2	A2	5.08
23	A2	A1	A2	A2	A1	5.09
19	A2	A1	A1	A2	A1	5.11
13	A1	A2	A2	A1	A1	5.19
24	A2	A1	A2	A2	A2	5.19
20	A2	A1	A1	A2	A2	5.21
9	A1	A2	A1	A1	A1	5.27
14	A1	A2	A2	A1	A2	5.3
10	A1	A2	A1	A1	A2	5.32
29	A2	A2	A2	A1	A1	5.32
25	A2	A2	A1	A1	A1	5.34
30	A2	A2	A2	A1	A2	5.43
26	A2	A2	A1	A1	A2	5.45
5	A1	A1	A2	A1	A1	5.52
1	A1	A1	A1	A1	A1	5.54
6	A1	A1	A2	A1	A2	5.62
2	A1	A1	A1	A1	A2	5.65
21	A2	A1	A2	A1	A1	5.66
17	A2	A1	A1	A1	A1	5.68
22	A2	A1	A2	A1	A2	5.77
18	A2	A1	A1	A1	A2	5.79

Existing

Soontorn's Integer DCT

Proposed

Existing

N	IR5	IR4	IR3	IR2	IR1	Average Sensitivity
14	A1	A2	A2	A1	A2	3.80
10	A1	A2	A1	A1	A2	3.82
16	A1	A2	A2	A2	A2	3.83
2	A1	A1	A1	A1	A2	3.83
12	A1	A2	A1	A2	A2	3.83
6	A1	A1	A2	A1	A2	3.84
30	A2	A2	A2	A1	A2	3.86
26	A2	A2	A1	A1	A2	3.86
4	A1	A1	A1	A2	A2	3.86
8	A1	A1	A2	A2	A2	3.87
22	A2	A1	A2	A1	A2	3.89
32	A2	A2	A2	A2	A2	3.89
18	A2	A1	A1	A1	A2	3.89
28	A2	A2	A1	A2	A2	3.89
24	A2	A1	A2	A2	A2	3.92
20	A2	A1	A1	A2	A2	3.92
13	A1	A2	A2	A1	A1	3.97
9	A1	A2	A1	A1	A1	3.97
15	A1	A2	A2	A2	A1	4.00
5	A1	A1	A2	A1	A1	4.00
1	A1	A1	A1	A1	A1	4.00
11	A1	A1	A2	A1	A2	4.00
29	A2	A2	A2	A1	A1	4.03
25	A2	A2	A1	A1	A1	4.03
3	A1	A1	A1	A2	A1	4.03
7	A1	A1	A2	A2	A1	4.04
31	A2	A2	A2	A2	A1	4.06
27	A2	A2	A1	A2	A1	4.06
21	A2	A1	A2	A1	A1	4.06
17	A2	A1	A1	A1	A1	4.06
23	A2	A1	A2	A2	A1	4.09
19	A2	A1	A1	A2	A1	4.09

Effectiveness for Fukuma's int-DCT

PSNR in Theory									
W [bit]	2	3	4	5	6	7	8	9	10
Existing	44.34	50.36	56.38	62.4	68.42	74.44	80.46	86.48	92.5
Proposed	45.82	51.84	57.86	63.88	69.9	75.92	81.94	87.96	93.98
Improvement (Average)	1.48 [dB]								

PSNR in Experiment									
W [bit]	2	3	4	5	6	7	8	9	10
Existing	41.78	48.78	54.02	63.68	67.75	75.84	81.76	89.47	90.57
Proposed	43.44	49.11	56.47	63.36	71.24	76.33	81.84	90.91	92.88
Improvement (Average)	1.33 [dB]								

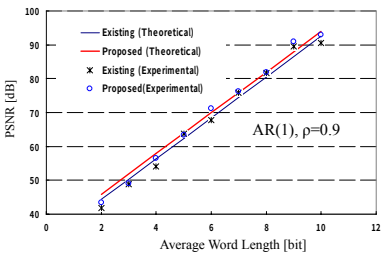


Image quality

Existing
PSNR=35.48 [dB]

Word length W = 3 [bit]
Fukuma's int-DCT

Proposed
PSNR=40.02 [dB]

Word length W = 3 [bit]
Fukuma's int-DCT

Fukuma's Int-DCT & W=3 [bit]

Image Name	Existing PSNR [dB]	Proposed PSNR [dB]	Improvement PSNR [dB]
barbara	35.48	40.02	4.54
aerial	35.37	38.76	3.39
lena	38.67	41.21	2.54
baboon	33.99	36.63	2.64
house	34.39	38.89	4.5
church	32.38	36.03	3.65
airplane	36.33	41.54	5.21
girl	42.19	45.71	3.52
		Average	3.75 [dB]

Soontorn's Int-DCT & W=3 [bit]

Image Name	Existing PSNR [dB]	Proposed PSNR [dB]	Improvement PSNR [dB]
barbara	42.03	44.85	2.82
aerial	41.07	44.36	3.29
lena	42.18	46.04	3.86
baboon	38.15	39.5	1.35
house	40.91	42.97	2.06
church	38.51	39.86	1.35
airplane	41.63	46.72	5.09
girl	48.7	50.32	1.62
		Average	2.68 [dB]

Comparison of the Two Algorithms

The best combination in ***Fukuma's***

Combination Number	IR5	IR4	IR3	IR2	IR1	Average Sensitivity
15	A1	A2	A2	A2	A1	4.67

The best combination in ***Soontorn's***

Combination Number	IR5	IR4	IR3	IR2	IR1	Average Sensitivity
14	A1	A2	A2	A1	A2	**3.8**