OPTIMUMIZATION OF LIFTING STRUCTRUE OF REVERSIBLE KLT BASED ON PERMUTATION OF SIGNAL'S ORDER AND SIGN

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ABSTRACT

This paper proposes a "reversible" three-point Karhunen Loeve transform (KLT) for de-correlation of RGB color components of an image signal. It is composed of "reversible" two point rotation transforms which have singular points (SP) and their rotation angles change depending on correlation of an input signal. When the angle is close to SP, rounding errors inside the transform are magnified to huge amount. To avoid this problem, we introduce permutation "order" and "sign" of signals. We also make it clear that the proposed method can shift SP by 0, 90, 180 or 270 degrees so that distance between SP and the rotation angle is maximized. It was observed that the proposed method reduces variance of the error by 10 [%].

Index Terms**—** error, KLT, reversible, coding

1. INTRODUCTION

The international standard of image or video signals such as JPEG or MPEG is composed of discrete cosine transform (DCT) [1]. It is well known that DCT is an asymptotic approximation of Karhunen Loeve transform (KLT) which is optimum for auto regressive (AR) model for de-correlation and source coding [2]. It has been also applied to color components of image signals for analysis [3-5].

These transforms are designed for implementation in fixed or floating point expression of signal values and coefficient values. In this case, rounding errors have been treated as negligible for "lossy" coding of image signals. However any kind of loss in a decoded signal is not permitted in "lossless" coding [6].

This report discusses how to reduce rounding errors generated inside a three-point "reversible" KLT which is applicable to lossless coding of color components. It is composed of two-point "reversible" rotation transforms (RT). Inside it, a signal value is rounded into an integer in the lifting structure [7]. It guarantees lossless reconstruction of original signals. However its output signals from the forward transform contain rounding errors.

In [8], permutation of order of two signals for the two point RT in DCT was introduced. It determined the best combination of the permutations to minimize errors due to rounding of coefficient values for each of image signals. In [9], permutation of order of three signals for the three point KLT was introduced. It was applied to lossless coding of color images as a reversible color transform.

In [10], existence of a singular point (SP) of the reversible two point rotation transform is discussed. As it pointed out, while a rotation angle changes depending on input signals, when the angle is close to the SP, rounding errors inside the transform are magnified to huge amount.

In this report, to cope with this SP problem, we introduce permutation of "order" and "sign" of signals in the two-point RT. We also make it clear that the proposed method can shift SP by 0, 90, 180 or 270 degrees so that distance between SP and the rotation angle is maximized. We experimentally confirm that the proposed method reduces variance of the error significantly.

2. EXISTING APPROACH

The singular point (SP) of a reversible rotation transform (RT) and permutation of signal's order are explained.

2.1. Reversible Rotation Transform

In this report, we discuss two types of two point rotation transforms:

$$\mathbf{G}(\theta)=\begin{bmatrix}\cos\theta & -\sin\theta\\ \sin\theta & \cos\theta\end{bmatrix},\ \mathbf{H}(\theta)=\begin{bmatrix}\cos\theta & \sin\theta\\ \sin\theta & -\cos\theta\end{bmatrix}. \quad (1)$$

$\mathbf{G}(\theta)$ and $\mathbf{H}(\theta)$ are referred to as Givens Jacobi Rotation and Householder Reflection respectively. Either of them can be factorized into

$$\begin{bmatrix}1 & 0\\ f_3 & 1\end{bmatrix}\begin{bmatrix}1 & f_2\\ 0 & 1\end{bmatrix}\begin{bmatrix}1 & 0\\ f_1 & 1\end{bmatrix}:=\mathbf{F}(\theta) \quad (2)$$

and implemented as illustrated in **Fig.1** which implies

$$\begin{bmatrix} x'_2 \\ y_1 \\ y_2 \end{bmatrix} = \begin{bmatrix} x_2 + R[f_1 x_1] \\ x_1 + R[f_2 x'_2] \\ x'_2 + R[f_3 y_1] \end{bmatrix}, \quad \begin{bmatrix} y'_2 \\ w_1 \\ w_2 \end{bmatrix} = \begin{bmatrix} y_2 - R[f_3 y_1] \\ y_1 - R[f_2 y'_2] \\ y'_2 - R[f_1 w_1] \end{bmatrix} \tag{3}$$

where $R[x]$ denotes rounding a value x into an integer. This procedure guarantees lossless reconstruction of an original signal at a decoder, namely $[w_1\ w_2]=[x_1\ x_2]$.

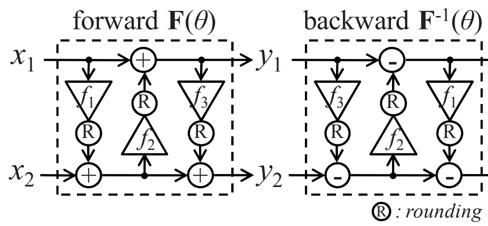

Fig.1 "Reversible" two point rotation transform (RT).

2.2. Singular Point of Reversible Rotation Transform

Coefficient values in Eq.(2) for $\mathbf{F}(\theta)=\mathbf{G}(\theta)$ are given by

$$\begin{cases} f_1 = f_3 = \tan(\theta/2) \\ f_2 = -\sin\theta \end{cases}. \tag{4}$$

It is obvious that when θ is close to $\pm\pi$ [rad], f_1 and f_3 are close to infinite. This is the singular point (SP) of the coefficients. In case of KLT, the rotation angle θ changes depending on correlation of input signals. If the angle is close to SP, output signals from the forward transform contains huge error even though a reconstructed signal from the backward transform has no error.

In this report, we evaluate the error with its variance. Denoting an error generated by the rounding just after f_m in **Fig.1** as e_m, errors in the output signals y_1 and y_2 are

$$\begin{bmatrix} e_{y1} \\ e_{y2} \end{bmatrix} = \begin{bmatrix} -\sin\theta & 1 & 0 \\ \cos\theta & \tan(\theta/2) & 1 \end{bmatrix} \begin{bmatrix} e_1 \\ e_2 \\ e_3 \end{bmatrix} \tag{5}$$

and their variances are

$$\begin{bmatrix} \sigma_{y1}^2 \\ \sigma_{y2}^2 \end{bmatrix} = \frac{1}{12} \begin{bmatrix} \sin^2\theta + 1 \\ \cos^2\theta + \tan^2(\theta/2) + 1 \end{bmatrix}. \tag{6}$$

As a result, total error is evaluated by

$$\sigma_{error}^2(\theta) = \sigma_{y1}^2 + \sigma_{y2}^2 = \frac{\tan^2(\theta/2)+3}{12}. \tag{7}$$

We can confirm that the error amount has SP at $\pm\pi$ [rad] and the rotation angle θ is expected to be far from this SP.

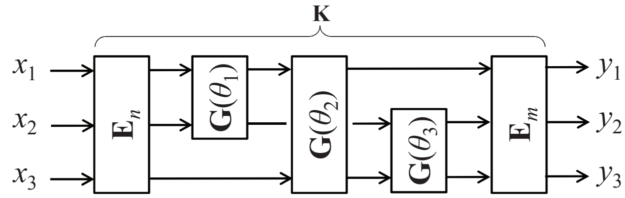

Fig.2 Three point KLT for color components.

2.3. Singular Point Avoidance by Permutation of Order

Fig.2 illustrates a three point reversible KLT for three components R,G,B of a color image. It has SP since $\mathbf{G}(\theta)$ in the figure is composed of the reversible RT in figure 1. In [9], permutation of order of three input signals was introduced. It is expressed as $\mathbf{E}_n$ and $\mathbf{E}_m$ in the figure and selected as one of the symmetric group:

$$\mathbf{Q}_1 = \begin{bmatrix} 1&0&0\\0&1&0\\0&0&1 \end{bmatrix}, \mathbf{Q}_2 = \begin{bmatrix} 0&1&0\\1&0&0\\0&0&1 \end{bmatrix}, \mathbf{Q}_3 = \begin{bmatrix} 1&0&0\\0&0&1\\0&1&0 \end{bmatrix},$$
$$\mathbf{Q}_4 = \begin{bmatrix} 0&1&0\\0&0&1\\1&0&0 \end{bmatrix}, \mathbf{Q}_5 = \begin{bmatrix} 0&0&1\\1&0&0\\0&1&0 \end{bmatrix}, \mathbf{Q}_6 = \begin{bmatrix} 0&0&1\\0&1&0\\1&0&0 \end{bmatrix}. \tag{8}$$

Since a basis matrix $\mathbf{K}$ of KLT is determined by covariance matrix of input signals $[x_1\ x_2\ x_3]$ as eigen vectors, when $\mathbf{K}$ is implemented as illustrated in **Fig.2**, it becomes

$$\mathbf{K} = \mathbf{E}_m \mathbf{Q}_5 \begin{bmatrix} \mathbf{G}(\theta_3) & 0 \\ 0 & 1 \end{bmatrix} \mathbf{Q}_6 \begin{bmatrix} \mathbf{G}(\theta_2) & 0 \\ 0 & 1 \end{bmatrix} \mathbf{Q}_3 \begin{bmatrix} \mathbf{G}(\theta_1) & 0 \\ 0 & 1 \end{bmatrix} \mathbf{E}_n \tag{9}$$
$$m, n \in \{1,2,3,4,5,6\}$$

and the rotation angles θ_1, θ_2, θ_3 changes depending on m and n for a given $\mathbf{K}$. In the existing method, the best lifting structure which has the least error amount is determined from all the $(3!)^2 = 36$ combinations.

In [9,10], "two"-input one-output lifting is introduced to reduce the number of rounding. However, since it sometimes magnify sensitivity of coefficients against errors, we focus on our discussion on one-input "one"-output lifting structure illustrated in Fig.1.

3. PROPOSED METHOD

Permutation of "sign" is additionally introduced. It increases freedom of avoidance of SP and reduce the error.

3.1. Introduction of Permutation of Sign

In this report, we introduce permutation of "sign" and "order" of two signals. These are applied to each of $\mathbf{G}(\theta_1)$, $\mathbf{G}(\theta_2)$, $\mathbf{G}(\theta_3)$ in the KLT. The permutation does not generate any error. The permutation of "sign" $\mathbf{S}$ and the permutation of "order" $\mathbf{P}$ are denoted as

$$\mathbf{P}=\begin{bmatrix}0 & 1\\ 1 & 0\end{bmatrix},\qquad \mathbf{S}=\begin{bmatrix}-1 & 0\\ 0 & 1\end{bmatrix}. \tag{10}$$

We apply them to **F** in Fig.1 as

$$\mathbf{G}(\theta)=\mathbf{P}_L\cdot\mathbf{F}(\psi)\cdot\mathbf{P}_R \tag{11}$$

where

$$\mathbf{P}_L\ ,\ \mathbf{P}_R\in\left\{\ \mathbf{P}^{s_1}\mathbf{S}^{s_2}\cdots\mathbf{P}^{s_{2t-1}}\mathbf{S}^{s_{2t}}\ \middle|\ \{s_u\}_{u\in\{1,2,\cdots\}}\in\mathbf{Z}\ \right\} \tag{12}$$

to construct a reversible rotation transform $\mathbf{G}(\theta)$ in Fig.2. Note that Eq.(12) includes all the combinations.

3.2. Four Candidates for Reversible Rotation Transform

Next, we show that it is enough to consider only four candidates for RT in Fig.2. Since permutation of order **P** and permutation of sign **S** have commutation relations:

$$\mathbf{P}^2=\mathbf{I},\quad \mathbf{S}^2=\mathbf{I},\quad (\mathbf{PS})^2=(-\mathbf{SP})^2=-\mathbf{I}, \tag{13}$$

Eq.(11) has a structure of dihedral group D4 described as:

$$\mathbf{P}_L\ ,\ \mathbf{P}_R\in\left\{\ (-\mathbf{I})^{s_1}\mathbf{P}^{s_2}\mathbf{S}^{s_3}\ \middle|\ s_1,s_2,s_3\in\{0,1\}\ \right\}. \tag{14}$$

Furthermore, since we focus on evaluation of error variance, negative sign can be ignored. As a result, the candidates are limited to

$$\mathbf{P}_L,\ \mathbf{P}_R\in\{\mathbf{I},\mathbf{P},\mathbf{S},\mathbf{PS}\}\ . \tag{15}$$

According to Eq.(1), inverse of the permutations in Eq.(10) and their product are expressed as

$$\mathbf{P}^{-1}=\mathbf{H}(\pi/2)\ ,\ \mathbf{S}^{-1}=\mathbf{H}(\pi)\ ,\ (\mathbf{PS})^{-1}=\mathbf{G}(\pi/2)\ . \tag{16}$$

The RT in Eq.(1) has mutual relations:

$$\begin{aligned}\mathbf{G}(\theta)\mathbf{G}(\varphi)&=\mathbf{G}(\theta+\varphi)\ ,\ \ \mathbf{H}(\theta)\mathbf{G}(\varphi)=\mathbf{H}(\theta-\varphi),\\ \mathbf{G}(\theta)\mathbf{H}(\varphi)&=\mathbf{H}(\theta+\varphi)\ ,\ \ \mathbf{H}(\theta)\mathbf{H}(\varphi)=\mathbf{G}(\theta-\varphi).\end{aligned} \tag{17}$$

Therefore, $(\mathbf{P}_L,\mathbf{P}_R)=(\mathbf{P},\mathbf{S})$ i.e. means:

$$\begin{aligned}\mathbf{F}(\theta)&=\mathbf{P}_L^{-1}\cdot\mathbf{G}(\theta)\cdot\mathbf{P}_R^{-1}=\mathbf{P}^{-1}\cdot\mathbf{G}(\theta)\cdot\mathbf{S}^{-1}\\ &=\mathbf{H}(\pi/2)\cdot\mathbf{G}(\theta)\cdot\mathbf{H}(\pi)=\mathbf{G}(-\theta-\pi/2)\end{aligned} \tag{18}$$

from Eq.(11),(16),(17). We exclude some cases from the candidates which do not satisfy the determinant conditions:

$$\begin{aligned}&|\,\mathbf{I}\,|=|\,\mathbf{PS}\,|=1,\ |\,\mathbf{P}\,|=|\,\mathbf{S}\,|=-1,\\ &|\,\mathbf{G}(\theta)\,|=|\,\mathbf{F}(\psi)\,|=1,\\ &|\,\mathbf{G}(\theta)\,|=|\,\mathbf{P}_L\,|\cdot|\,\mathbf{F}(\psi)\,|\cdot|\,\mathbf{P}_R\,|.\end{aligned} \tag{19}$$

As a result, the candidates are limited to only eight types as indicated in **Table I**. However some of them share the same SP. Therefore we can limit the candidates to the four cases described as

$$\begin{aligned}k=1:&\quad \mathbf{G}(\theta)=\mathbf{F}(\psi),\\ k=2:&\quad \mathbf{G}(\theta)=\mathbf{P}\cdot\mathbf{F}(\psi)\cdot\mathbf{S},\\ k=3:&\quad \mathbf{G}(\theta)=\mathbf{S}\cdot\mathbf{F}(\psi)\cdot\mathbf{P},\\ k=4:&\quad \mathbf{G}(\theta)=\mathbf{PS}\cdot\mathbf{F}(\psi)\cdot\mathbf{PS}.\end{aligned} \tag{20}$$

Table I The angle ψ of $\mathbf{F}(\psi)$ in Eq.(20).

		$\mathbf{P}_R$			
		I	**P**	**S**	**PS**
$\mathbf{P}_L$	**I**	θ $_{(k=1)}$	---	---	$\theta+\pi/2$
	P	---	$-\theta$	$-\theta-\pi/2$ $_{(k=2)}$	---
	S	---	$-\theta+\pi/2$ $_{(k=3)}$	$-\theta$	---
	PS	$\theta+\pi/2$	---	---	$\theta+\pi$ $_{(k=4)}$

"---" : not available

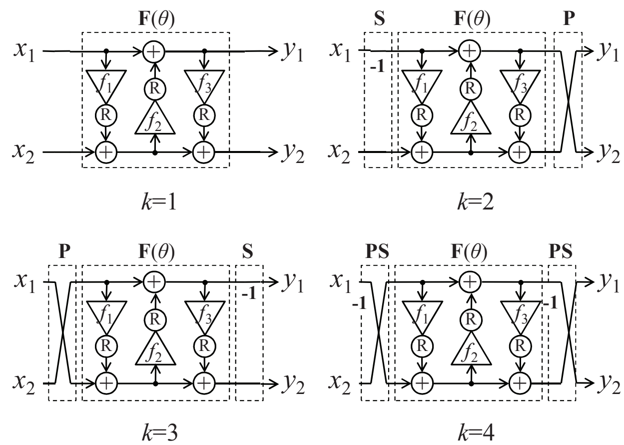

Fig.3 Four candidates for $\mathbf{G}(\theta)$ in the reversible KLT.

3.3. Determination of the Best Combination

In the proposed method, one of four candidates in Eq.(20) also illustrated in **Fig.3** is determined for each of $\mathbf{G}(\theta_1)$, $\mathbf{G}(\theta_2)$, $\mathbf{G}(\theta_3)$ in **Fig.2**. As summarized in table I, these are expressed in the form of Givens Jacobi rotation as:

$$\begin{cases}k=1 & \mathbf{F}(\psi)=\mathbf{G}(+\theta) & \theta_{SP}=\pi\\ k=2 & \mathbf{F}(\psi)=\mathbf{G}(-\theta-\pi/2) & \theta_{SP}=\pi/2\\ k=3 & \mathbf{F}(\psi)=\mathbf{G}(-\theta+\pi/2) & \theta_{SP}=3\pi/2\\ k=4 & \mathbf{F}(\psi)=\mathbf{G}(+\theta-\pi) & \theta_{SP}=0\end{cases}. \tag{21}$$

where θ_{SP} indicates SP of each candidate. As a result of introducing permutation of "sign", it became possible to shift SP by 0, 90, 180 or 270 degrees.

It is also interesting to see that it is not necessary to try all the four combinations for each of three RTs. It requires 4^3 calculation of error amount. Instead, according to Eq.(21), we just need to pick up one of the candidates which has the minimum distance between a given rotation angle and the SP. It requires only 3 calculations of the distance.

4. EXPERIMENTAL RESULTS

We evaluate variance of errors of the reversible KLT applying to RGB components of color image signals.

Table II summarizes rotation angles calculated as eigen vector of covariance matrix of each input image. **Table III** summarizes lifting structure of each of RT in the KLT. For example for Lena, the existing method has [θ_1 θ_2 θ_3]= [30.6 39.5 -110.8] degrees and all the RT are limited to k=1 in Fig.3.

On the contrary, in the proposed method, rotation angles are [θ_1 θ_2 θ_3]= [-130.1 -19.3 45.3] and each of RT is constructed as [k_1 k_2 k_3]=[3 1 1]. We can confirm that result of the 4^3 error amount calculation is exactly the same as that of the 4 distance calculation. It is confirmed that our analysis reduces optimization procedure of finding the best lifting structure of the candidates illustrated in Fig.3.

Table IV summarizes total error and bit rate. It was observed that the proposed method reduces variance of the error by approximately 10 [%] in average. The bit rate of {R, G, B} is reduced from 24 to 12.7 [bpp] with the reversible KLT, 5/3 lossless wavelet and EBCOT. There is no significant difference between the two methods.

Table II Rotation angles of KLT for each image. [degree]

input	existing			proposed		
	θ_1	θ_2	θ_3	θ_1	θ_2	θ_3
Lena	30.6	39.5	-110.8	-130.1	-19.3	45.3
Sailboat	77.6	46.6	-43.8	-112.5	-28.4	55.7
Ballon	80.9	46.6	-55.3	-131.5	-41.0	81.7
Earth	29.0	27.5	-108.3	-120.0	-13.4	37.1

Table III Optimum lifting structure for each image.

input	existing			proposed		
	k_1	k_2	k_3	k_1	k_2	k_3
Lena	1	1	1	**3**	1	1
Sailboat	1	1	1	**3**	1	**2**
Ballon	1	1	1	**4**	**3**	**4**
Earth	1	1	1	**4**	1	1

Table IV Performance of the KLT for each image.

input	variance of rounding error		PSNR[dB]		bit rate [bpp] (R,G,B)	
	existing	proposed	existing	proposed	existing	proposed
Lena	0.31	0.26	53.17	54.03	13.49	13.49
Sailboat	0.28	0.26	53.70	54.03	13.50	13.51
Ballon	0.28	0.26	53.63	53.94	10.95	10.95
Earth	0.31	0.26	53.22	53.99	12.64	12.66
average	0.29	0.26	53.43	54.00	12.65	12.65

5. CONCLUSIONS

In this paper, we proposed a "reversible" Karhunen Loeve transform (KLT) which can reduce rounding error generated inside the transform. Applying it to de-correlation of color components of image signals, it was observed that the proposed method reduces variance of the error by approximately 10 [%] in average. As a result of our analysis, we found that the proposed method shifts the singular point (SP) by 0, 90, 180 or 270 degrees so that distance between the SP and the rotation angle of the KLT is maximized. It contributes to reduce complexity of optimization procedure of the lifting structure.

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