Lossless Coding of Still Images with Four Channel Prediction

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Abstract

In this paper a new lossless coding of digital image data with four channel filter bank (4ch FB) having optimized filter coefficients is proposed. The method has an advantage of less restrictions on optimizing filter coefficients comparing to cascade form of two channel filter banks (2ch FB). Simulation results are shown to confirm its effectiveness.

1 Introduction

It is well known that image data have strong correlation among pixels in general. This characteristics is effectively utilized when data amount of the image is compressed. Especially when we need lossless coding, differential pulse code modulation (DPCM) is widely used [1]. Recently lossless codings based on two channel filter bank (2ch FB) are proposed [2]. However performance of these existing methods is limited by a restriction due to cascading the 2ch FB.

In this report we propose a new lossless coding which can be optimized for arbitrary input image data. Our method is based on four channel filter bank (4ch FB) free from the restriction of the 2ch FB. We optimize filter coefficients of the FB and confirm effectiveness of our method using some image data.

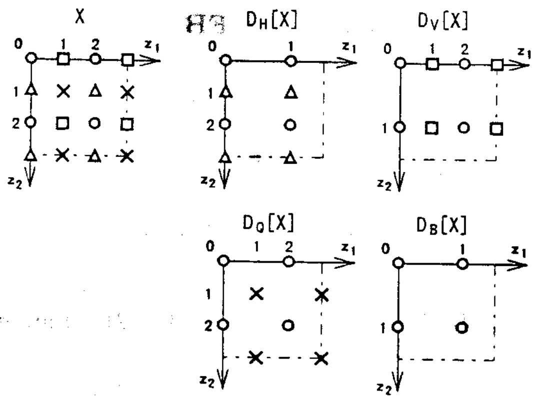
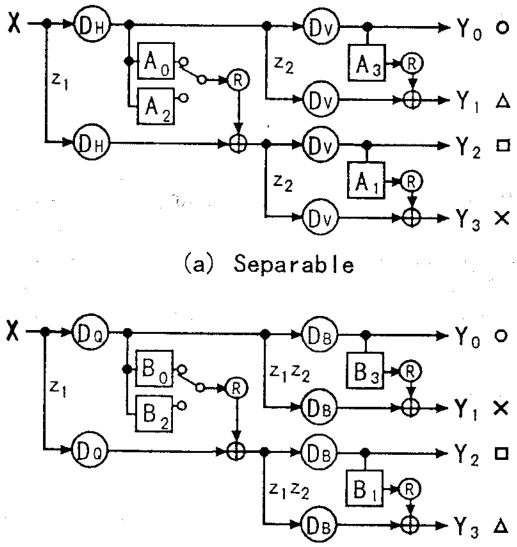


Figure 1: Sub samplings of 2D signal.

2 Two Channel Predictions

2.1 Cascade of 2ch FB

Lossless encoding of an image signal X based on the 2ch FB [2] is illustrated in figure 2(a),(b). "R" in the figure denotes a rounding of a real value into an integer. Sub samplings in the figure are shown in figure 1. We deal with the case that X and filter coefficients of the FB are expressed with 8 bit integer and real values respectively. We also focus attention on splitting X into four band signals Y_i,(i=0,1,2,3).



(a) Separable

(b) Non separable

Figure 2: Cascade of the 2ch FB.

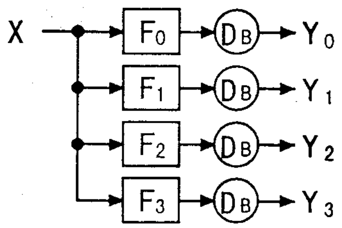


Figure 3: Generalized form of the FB.

A_i,(i=0,1,2,3) in figure 2(a) are all 1D filters therefore this FB effectes on X as a *separable* 2D filter. On the other hand B_i,(i=0,1,2,3) in figure 2(b) are 2D *non separable* filters.

2.2 Restrictions on the FB

Separable FB in figure 2(a) can be expressed as the generalized 4ch FB in figure 3 with F_i,(i=0,1,2,3) given by

$$\begin{bmatrix} F_0(z_1,z_2) \\ F_1(z_1,z_2)z_2^{-1} \\ F_2(z_1,z_2)z_1^{-1} \\ F_3(z_1,z_2)z_1^{-1}z_2^{-1} \end{bmatrix} = \begin{bmatrix} 1 \\ A_3'(z_2) \\ A_2'(z_1) \\ A_0'(z_1)A_1'(z_2) \end{bmatrix} \quad (1)$$

where

$$A_i'(z_j) = 1 + A_i(z_j^2)z_j^{-1}, \quad j = (i \ mod \ 2) + 1.$$

For non separable FB, F_i,(i=0,1,2,3) are

$$\begin{bmatrix} F_0(z_1,z_2) \\ F_1(z_1,z_2)z_2^{-1} \\ F_2(z_1,z_2)z_1^{-1} \\ F_3(z_1,z_2)z_1^{-1}z_2^{-1} \end{bmatrix} = \begin{bmatrix} 1 \\ B_3'(z_1,z_2) \\ B_2'(z_1,z_2) \\ B_0'(z_1,z_2)B_1'(z_1,z_2) \end{bmatrix} \quad (2)$$

where

$$B_i'(z_1,z_2) = 1+B_i(z_1^j,z_2^j)z_1^{-1}z_2^{-j+1}, \quad j = (i \ mod \ 2)+1.$$

As indicated in the last column of eq.(1) (or eq.(2)) Frequency amplitude characteristics $|F_3|$ must be a product of two functions $|A_0'|$ and $|A_1'|$ (or $|B_0'|$ and $|B_1'|$). This is the restriction on optimizing the FB. We propose 4ch FB free from this restriction next.

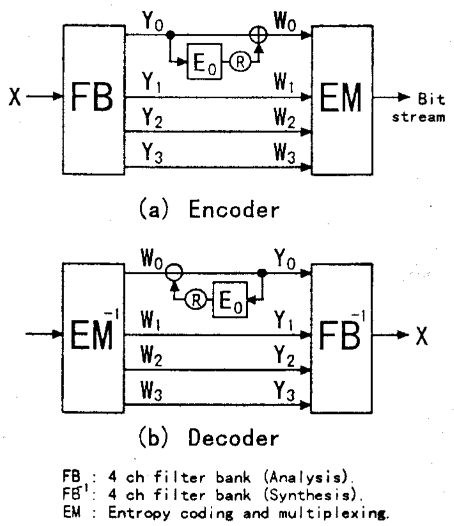


Figure 4: Proposed coding scheme.

3 Four Channel Prediction

3.1 Encoding and Decoding

We propose the coding scheme illustrated in figure 4. In this figure we use the 4ch FB in fugure 5. Equivalent FB in figure 3 is given by

$$\begin{bmatrix} F_0(z_1,z_2) \\ F_1(z_1,z_2)z_1^{-1}z_2^{-1} \\ F_2(z_1,z_2)z_1^{-1} \\ F_3(z_1,z_2)z_2^{-1} \end{bmatrix} = \begin{bmatrix} 1 \\ 1+C_5' \\ 1+C_3'+C_4' \\ 1+C_0'+C_1'+C_2' \end{bmatrix} \quad (3)$$

where

$$C_i' = C_i(z_1^2,z_2^2)z_1^{-j}z_2^{-k}$$

and (i,j,k)=(0,1,0), (1,0,1), (2,1,1), (3,1,0), (4,1,-1), (5,0,1).

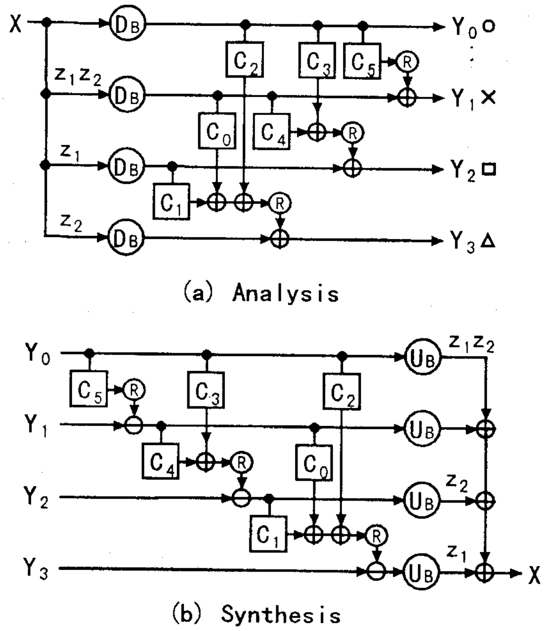


Figure 5: The four channel FB.

3.2 Restrictions on the FB

As an example we consider the case of using neighboring 3x3 pixels in the filtering. Using the matrix

$$\mathbf{Z} = \begin{bmatrix} 1 \\ z_1 + z_1^{-1} \\ z_2 + z_2^{-1} \\ z_1z_2 + z_1^{-1}z_2^{-1} \\ z_1z_2^{-1} + z_1^{-1}z_2 \end{bmatrix} \quad (4)$$

A_i', B_i', C_i',(i=0,1,$\cdots$) in equations (1), (2),(3) are expressed as follows.

$$\begin{bmatrix} A_0'(z_1,z_2) \\ A_1'(z_1,z_2) \\ A_2'(z_1,z_2) \\ A_3'(z_1,z_2) \end{bmatrix} = \begin{bmatrix} 1 & a_0 & 0 & 0 & 0 \\ 1 & 0 & a_1 & 0 & 0 \\ 1 & a_2 & 0 & 0 & 0 \\ 1 & 0 & a_3 & 0 & 0 \end{bmatrix} \mathbf{Z} \quad (5)$$

$$\begin{bmatrix} B'_0(z_1,z_2) \\ B'_1(z_1,z_2) \\ B'_2(z_1,z_2) \\ B'_3(z_1,z_2) \end{bmatrix} = \begin{bmatrix} 1 & b_0 & b_1 & 0 & 0 \\ 1 & 0 & 0 & b_2 & b_3 \\ 1 & b_4 & b_5 & 0 & 0 \\ 1 & 0 & 0 & b_6 & b_7 \end{bmatrix} \mathbf{Z} \qquad (6)$$

$$\begin{bmatrix} C'_0(z_1,z_2) \\ C'_1(z_1,z_2) \\ C'_2(z_1,z_2) \\ C'_3(z_1,z_2) \\ C'_4(z_1,z_2) \\ C'_5(z_1,z_2) \end{bmatrix} = \begin{bmatrix} 0 & c_0 & 0 & 0 & 0 \\ 0 & 0 & c_1 & 0 & 0 \\ 0 & 0 & 0 & c_2 & c_3 \\ 0 & c_4 & 0 & 0 & 0 \\ 0 & 0 & 0 & c_5 & 0 \\ 0 & 0 & c_7 & 0 & c_6 \end{bmatrix} \mathbf{Z} \qquad (7)$$

In this case $|F_3|$ in equations (1),(2),(3) become

$$\begin{bmatrix} |F_3|\ in\ (1) \\ |F_3|\ in\ (2) \\ |F_3|\ in\ (3) \end{bmatrix} = \begin{bmatrix} 1 & a_0 & a_1 & a_0a_1 & a_0a_1 & 0 & 0 & 0 & 0 \\ 1 & b_0+b_1(b_2+b_3) & b_1+b_0(b_2+b_3) & b_2 & b_3 & b_0b_2 & b_0b_3 & b_1b_2 & b_1b_3 \\ 1 & c_0 & c_1 & c_2 & c_3 & 0 & 0 & 0 & 0 \end{bmatrix} \begin{bmatrix} 1 \\ 2\cos\omega_1 \\ 2\cos\omega_2 \\ 2\cos(\omega_1+\omega_2) \\ 2\cos(\omega_1-\omega_2) \\ 2\cos(2\omega_1+\omega_2) \\ 2\cos(2\omega_1-\omega_2) \\ 2\cos(\omega_1+2\omega_2) \\ 2\cos(\omega_1-2\omega_2) \end{bmatrix} \qquad (8)$$

Comparing these three methods it becomes clear that the parameters c_i,(i=0,1,$\cdots$,7) are independent of each others. Therefoer the 4ch FB has less restrictions than other 2ch FBs including [3, 4].

3.3 Optimization of the FB

Assuming that bit rate B_i of each band signal Y_i is in propotion to logarithm of its variance σ^2_{Yi}, namely,

$$B_i = \epsilon \log_2 \sigma^2_{Yi} \qquad (9)$$

total bit rate $\bar{B}$ become

$$\bar{B} = \frac{1}{4}\sum_{i=0}^{3} B_i = \frac{\epsilon}{4}\log_2 I \qquad (10)$$

where

$$I = \prod_{i=0}^{3} \sigma^2_{Yi} \qquad (11)$$

To optimize filter coefficients c_i,(i=0,1,$\cdots$,7) in equation (7) we minimize each of σ^2_{Yi} in equation (11). This problem can be solved using Yule Walker's equation as a linear optimization problem. This is because each of $|F_i|$,(i=0,1,2,3) in our method is expressed as a linear combination of unknown parameters c_i.

4 Simulation

As results of performing our method, variance of the band signal Y_i, (i=0,1,2,3) are minimized. Optimized predictions are illustrated in figure 7. Variance gain G_i of each band defined by

$$G_i = 10\log_{10}\frac{\sigma^2_{Wi}}{\sigma^2_X}. \qquad (12)$$

are summarized in table 1 where σ^2_X and σ^2_{Wi} are variance of X and W_i respectively. Notice that we used the extrapolation

$$E_0(z_1,z_2) = 1 - \frac{z_1^{-1}+z_2^{-1}}{2} \qquad (13)$$

on Y_0 to get W_0 and we used 256x256 pixels of standard image data. Entropys of W_i are summarized in table 2. We can confirm superiority of our method to the DPCM (Equation (13) is applied on X directly.) by table 1 and table 2. Levels (range of values) and frequency characteristics are indicated in table 3 and figure 7 respectively as auxiliary information.

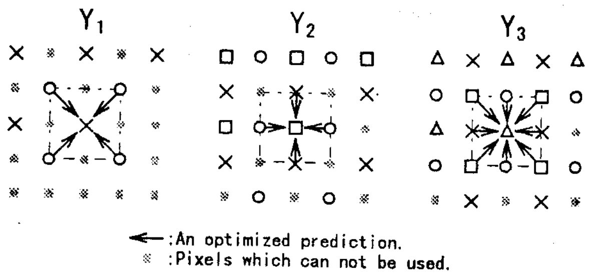


Figure 6: Optimized predictions in each band.

5 Summary and Conclusions

We proposed a new lossless coding of digital image data based on 4ch FB and optimized its filter coefficients. Our method has an advantage over cascading 2ch FB that the coefficients are obtained by solving linear equations. We confirmed superiority of our method to the DPCM compressing some image data.

References

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[2] K.Komatsu, K.Sezaki, "Reversible Transform Coding of Images", IEICE Trans. of Fundamentals, vol.J79-A, No.4, pp.981-990 (April 1996)

[3] A.Said, W.A.Pearlman, "An Image Multiresolution Representaton for Lossless and Lossy Compression", IEEE Trans. on Image Processing, vol.5, No.9, pp.1303-1310 (Sept. 1996)

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Table 1. Gain [dB]

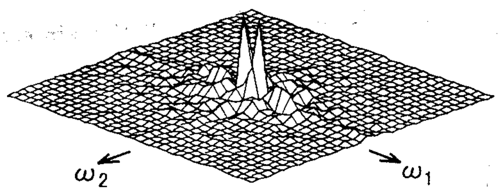
	PCM	DPCM	Proposed				
			Band 0	Band 1	Band 2	Band 3	Average
Barbara	0.0000	-8.4389	-5.7636	-8.9450	-17.4749	-19.0004	-12.7960
Couple	0.0000	-21.4032	-17.2402	-20.9525	-24.4930	-27.0590	-22.4362
Girl	0.0000	-18.5146	-14.3166	-18.8062	-21.7950	-22.4537	-19.3429
Lena	0.0000	-13.8705	-9.2808	-15.2850	-18.8170	-19.9509	-15.8334
Basketball	0.0000	-10.5836	-7.2256	-10.7223	-13.7581	-14.4987	-11.5512
Flower	0.0000	-10.8695	-8.1816	-10.0050	-17.3860	-17.6923	-13.3162
Mobile	0.0000	-7.4199	-4.8805	-7.8530	-9.4365	-10.2983	-8.1171

Table 2. Entropy [bit/pixel]

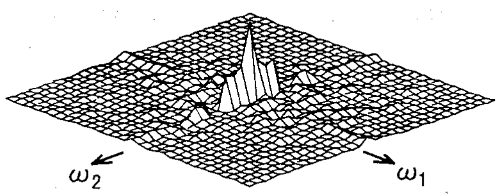
	PCM	DPCM	Proposed				
			Band 0	Band 1	Band 2	Band 3	Average
Barbara	7.4617	5.7201	6.2122	5.5999	4.4554	4.3322	5.1499
Couple	6.2200	4.5658	5.1963	4.6810	4.1630	3.8124	4.4632
Girl	6.4145	4.7888	5.3783	4.7239	4.3844	4.3337	4.7051
Lena	7.4902	5.1323	5.8473	4.9885	4.4763	4.3288	4.9102
Basketball	7.6678	5.9339	6.4736	5.9729	5.5001	5.3608	5.8269
Flower	7.5231	5.7574	6.0726	5.9765	4.6088	4.6133	5.3178
Mobile	7.5260	6.0949	6.5306	6.1851	5.7579	5.6275	6.0253

Table 3. Levels

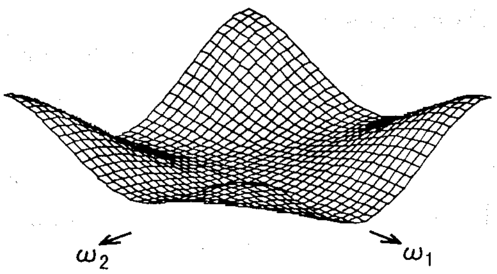
	PCM	DPCM	Proposed				
			Band 0	Band 1	Band 2	Band 3	Average
Barbara	220	196	237	244	73	83	159.3
Couple	240	178	239	145	94	98	144.0
Girl	254	182	282	169	103	87	160.3
Lena	210	184	261	139	90	104	148.5
Basketball	254	288	369	234	197	244	261.0
Flower	239	286	364	273	142	134	228.3
Mobile	238	370	426	296	309	265	324.0



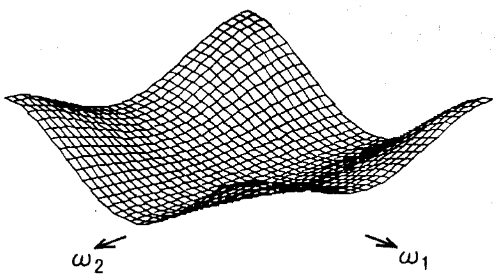
(a) |X| of "Barbara".



(b) |X| of "Basketball".



(c) $|F_3|$ for "Barbara".



(d) $|F_3|$ for "Basketball".

Figure 7: Frequency amplitude characteristics.