Two Channel Non-Separable 2D Subband Coding and Its Optimization

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Abstract

In this paper, a new lossy/lossless coding of digital image data with two channel non separable filter bank is proposed. The method has freedom of selecting taps and coefficients of the filters and this freedom is used for maximizing the coding gain. Simulation results are shown to confirm its effectiveness.

1 Introduction

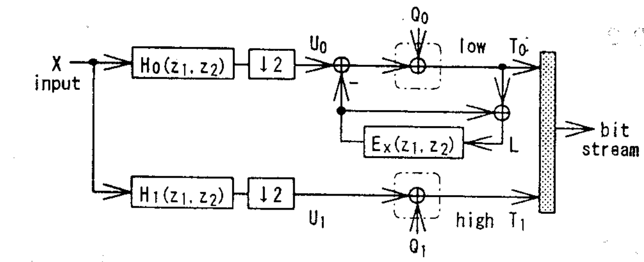
Discrete cosine transform (DCT) is one of the most widely used technique as a generic coding of images [1, 2, 3]. However the DCT is not always optimum for arbitrary input image data. In case of the optimization is necessary, Karhunen - Loeve transform (KLT) may be used at the sacrifice of large number of parameters, i.e. coefficients of basis functions [4]. Recently lapped orthogonal transform (LOT) and two channel filter banks respectively in [5] and [6, 7] are optimized for input image data under the measure called coding gain [4]. However these filters are used as *separable* filters for image data and these separable filters have restrictions on maximizing coding gain because they have less freedom of parameters than *non separable* ones. Non separable filter banks are found in [8, 9, 10] and they have fixed number of taps, i.e. 3x3 and 5x5 taps, and fixed values of coefficients. This is the problem to be discussed here.

In this report we propose a new type of non separable two dimensional (2D) filter bank which has freedom of choice of taps, i.e. 3x3, 5x5, 7x7, 9x9, etc., and coefficient values. To begin with, we mention the problem of the existing filter banks. Next, we develop the new non-separable filter bank satisfying perfect reconstruction requirement and we optimize filter coefficients of the proposed filter bank to maximize the unified coding gain [8]. Furthermore, we extend our method to lossy/lossless coding. Namely, our method can be used as a lossy coding and also a lossless coding. Finally, we confirm effectiveness of the proposed method using some image data.

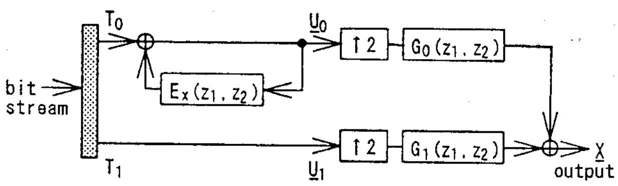
2 Non Separable 2D subband Coding

2.1 Coding and Decoding

Figure 1 illustrates a subband coding procedure with two channel filter bank. In this figure, input signal X is filtered with H_i,(i=0,1) and sub sampled as



(a) Encoder



(b) Decoder

Figure 1: Two channel subband coding.

shown in figure 2 to produce band signals U_i,(i=0,1). Namely,

$$U_i(z_1, z_2) = D[Y_i(z_1, z_2)], \ (i = 0, 1) \tag{1}$$

where

$$Y_i(z_1, z_2) = H_i(z_1, z_2)X(z_1, z_2) \tag{2}$$

and

$$D[Y_i(z_1, z_2)] = \frac{Y_i(z_1^{1/2}, z_2^{1/2}) + Y_i(-z_1^{1/2}, -z_2^{1/2})}{2} \tag{3}$$

After above processing, U_0 are extrapolated with

$$E_x(z_1, z_2) = \frac{z_1^{1/2} + z_1^{-1/2}}{2} z_2^{-1/2} \tag{4}$$

Then both of U_0 and U_1 are quantized following entropy coding and multiplexing to be transmitted to the decoder as a bitstream.

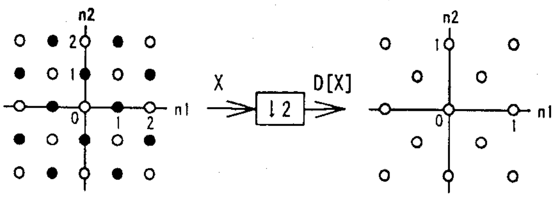


Figure 2: Sub sampling of 2D signal.

In the decoder, the bit stream is expanded in inverse manner of the encoding. To satisfy perfect reconstruction requirment [9] G_i,(i=0,1) are designed by

$$\begin{bmatrix} G_0(z_1,z_2) \\ G_1(z_1,z_2) \end{bmatrix} = \begin{bmatrix} -H_1(-z_1,-z_2) \\ H_0(-z_1,-z_2) \end{bmatrix} \quad (5)$$

2.2 Problem of The Existing Method

Non separable filters in [8, 9, 10] are expressed by

$$\begin{bmatrix} H_0(z_1,z_2) \\ H_1(z_1,z_2) \end{bmatrix} = \begin{bmatrix} 1 & 0 \\ 0 & z_1^{-1} \end{bmatrix} \cdot$$

$$\begin{bmatrix} \frac{7}{8} & \frac{1}{8} & \frac{-1}{16} & \frac{-1}{32} \\ 1 & \frac{-1}{4} & 0 & 0 \end{bmatrix} \begin{bmatrix} 1 \\ z_1+z_1^{-1}+z_2+z_2^{-1} \\ (z_1+z_1^{-1})(z_2+z_2^{-1}) \\ z_1^2+z_1^{-2}+z_2^2+z_2^{-2} \end{bmatrix} \quad (6)$$

As indicated above, this filter has fixed number of taps (i.e. H_0 is 5x5 tap and H_1 is 3x3 tap) and fixed values of coefficients. Therefore the existing method is not always optimum for arbitrary input image data in respect of data compression.

3 Optimization

3.1 Unified Coding Gain

Assuming that each of T_i,(i=0,1) in figure 1 is expressed with B_i [bit/pixel], optimized bit allocation under the average bit rate B_{ave} [bit/pixel] is given by

$$B_{k,opt} = B_{ave} + \frac{1}{2}\log_2 \frac{\|E_k H_k X\|^2 \|G_k\|^2}{\prod_{j=0}^{1}\{\|E_j H_j X\|^2 \|G_j\|^2\}^{\frac{1}{2}}} \quad (7)$$

where $k = 0, 1$ and

$$E_k(z_1,z_2) = \begin{cases} 1 - E_x(z_1^2, z_2^2) & (k=0) \\ 1 & (k=1) \end{cases}$$

and $\|X\|$ denotes variance of X. In this case coding efficiency can be evaluated by

$$G_{SBC} = 10\log_{10} \frac{\|X\|^2}{\prod_{j=0}^{1}\{\|E_j H_j X\|^2 \|G_j\|^2\}^{\frac{1}{2}}} \quad (8)$$

This is an extended version of *the unified coding gain* in [8] considering extrapolation E_x. In this paper we determine filter coefficients of H_0,H_1,G_0,G_1 to maximize G_{SBC} under the perfect reconstruction [9].

3.2 Perfect Reconstruction

Output signal $\underline{X}$ in figure 1 is given by

$$\underline{X}(z_1,z_2) = \frac{1}{2}\sum_{i=0}^{1} T_i(z_1,z_2) X((-1)^i z_1, (-1)^i z_2) \quad (9)$$

where

$$T_i(z_1,z_2) = \sum_{j=0}^{1} (-1)^{ij} H_j((-1)^i z_1, (-1)^i z_2) G_j(z_1,z_2) \quad (10)$$

Substituting equation (5) into equation (10) we get

$$\begin{bmatrix} T_0(z_1,z_2) \\ T_1(z_1,z_2) \end{bmatrix} = \begin{bmatrix} 0 \\ H_0^- H_1^+ - H_0^+ H_1^- \end{bmatrix} \quad (11)$$

where

$$H_k^{\pm} = H_k(\pm z_1, \pm z_2), \quad k = 0, 1 \quad (12)$$

Then the perfect reconstruction constrain requires T_1 having all pass characteristics.

4 Proposed Method

4.1 Structural Condition

To have freedom of selecting taps and coefficients of the filter bank under the perfect reconstruction constrain, we extend 1D linear phase two channel filter bank in [11] to non separable 2D filter bank. Namely, we use

$$H_0(z_1,z_2) = 1 - F(z_1,z_2) H_1(z_1,z_2) \quad (13)$$

as a low pass filter. In this case substituting equation (13) into equation (11) we get

$$T_1(z_1,z_2) = H_1^+ - H_1^- + (F^+ - F^-) H_1^+ H_1^- \quad (14)$$

where

$$F^{\pm} = F(\pm z_1, \pm z_2)$$

Therefore whenever

$$\begin{bmatrix} H_1^+ & H_1^- \\ F^+ & F^- \end{bmatrix} \begin{bmatrix} 1 \\ -1 \end{bmatrix} = \begin{bmatrix} 2z_1^{-1} \\ 0 \end{bmatrix} \quad (15)$$

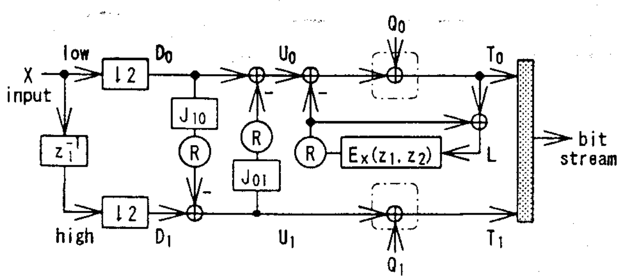
is satisfied the filter bank satisfies the perfect reconstruction constrain. In this paper we determine taps and coefficients of H_0 and H_1 under equations (5),(13),(15) so that the unified coding gain G_{SBC} is maximized [12].

4.2 Lossy/Lossless Coding

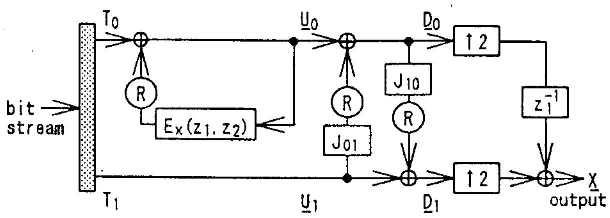
To attain lossy/lossless coding we use encoder and decoder in the figure 3 where

$$\begin{bmatrix} F(z_1^{1/2}, z_2^{1/2}) \\ z_1^{-1/2} - H_1(z_1^{1/2}, z_2^{1/2}) \end{bmatrix} = \begin{bmatrix} J_{01} \\ J_{10} \end{bmatrix} \quad (16)$$

"R" in this figure denotes rounding a real value into an integer. The lossless coding of images is attained when the step size of the quantization is equals to one.



(a) Encoder



(b) Decoder

Figure 3: Proposed lossy/lossless coding.

5 Simulation

In our method J_{01} and J_{10} are expressed as follows.

$$\begin{bmatrix} J_{01} \\ J_{10} \end{bmatrix} = \begin{bmatrix} z_1^{1/2} & 0 \\ 0 & z_1^{-1/2} \end{bmatrix} \begin{bmatrix} a_0 & a_1 & a_2 & a_3 & \dots \\ b_0 & b_1 & b_2 & b_3 & \dots \end{bmatrix} \cdot$$

$$\begin{bmatrix} z_1^{1/2} + z_1^{-1/2} \\ z_2^{1/2} + z_2^{-1/2} \\ (z_1 + z_1^{-1})(z_2^{1/2} + z_2^{-1/2}) \\ (z_1^{1/2} + z_1^{-1/2})(z_2 + z_2^{-1}) \\ \vdots \end{bmatrix} \qquad (17)$$

Optimized coefficients for "Barbara (256x256 pixels)" are summarized in table 1 and table 2. Values of the unified coding gain for some digital image data are indicated in table 3. Effectiveness of our method over existing method can be confirmed by 3 [dB] for "Barbara". Rate distortion curves are illustrated in figure 4.

Performance of lossless mode is evaluated using entropy defined by

$$H = -\sum_s P_s \log_2 P_s \qquad (18)$$

where P_s is a probability of symbol "s". Effectiveness of our method over existing methods including DPCM

$$E_{DPCM}(z_1, z_2) = 1 - \frac{z_1^{-1} + z_2^{-1}}{2} \qquad (19)$$

can also be confirmed for image data (256x256 pixels).

6 Summary and Conclusions

We proposed a non-separable 2D filter bank having freedom of choice of taps and coefficients under perfect reconstruction requirement. Furthermore we optimized the coefficients to maximize the unified coding gain for each input image data and confirmed effectiveness of the proposed method.

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Table 1: Filter coefficients of J01.

	a0	a1	a2	a3
Existing (5x3)	-0.1250	-0.1250	---	---
Proposed (1x3)	---	---	---	---
Proposed (1x5)	---	---	---	---
Proposed (5x3)	0.0328	-0.2782	---	---
Proposed (7x3)	0.0400	-0.2803	0.0000	-0.0199
Proposed (7x5)	-0.0391	-0.2881	---	---

Table 2: Filter coefficients of J10.

	b0	b1	b2	b3
Existing (5x3)	0.1250	0.1250	---	---
Proposed (1x3)	-0.0078	0.5078	---	---
Proposed (1x5)	0.0985	0.5286	-0.0195	-0.0440
Proposed (5x3)	-0.0568	0.5568	---	---
Proposed (7x3)	-0.0584	0.5584	---	---
Proposed (7x5)	0.07986	0.5764	-0.0133	-0.0648

Table 3: Coding gain [dB].

	Barbara	Mobile	Flower
Existing (5x3)	8.4053	7.1000	9.4755
Proposed (1x3)	10.4379	7.1848	10.8500
Proposed (1x5)	10.7471	7.1964	10.8651
Proposed (5x3)	11.1346	7.6309	11.5620
Proposed (7x3)	11.1429	7.6341	11.5625
Proposed (7x5)	11.5558	7.7759	11.6552

Table 4: Entropy [bit/pixel].

	Barbara	Mobile	Flower
PCM	7.4617	7.5260	7.5231
DPCM	5.7219	6.1019	5.7634
Existing (5x3)	5.6544	6.1346	5.9140
Proposed (1x3)	5.2611	6.0279	5.4710
Proposed (1x5)	5.2048	6.0265	5.4704
Proposed (5x3)	5.2478	6.0060	5.4397
Proposed (7x3)	5.2470	6.0056	5.4396
Proposed (7x5)	5.2082	6.0199	5.4501

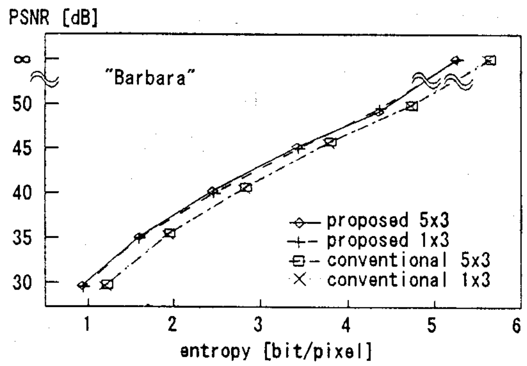

Figure 4: Rate distortion curves.