Lossless / Lossy Progressive Coding based on Reversible Wavelet and Lossless Multi-channel Prediction

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ABSTRACT

In this report, a digital image data compression method which has two modes – *"losslesss"* mode and *"lossy"* mode – and also *"progressive"* functionality is proposed. The coding system can be optimized for an arbitrary input image signal under the lossless / lossy unified coding gain introduced here.

1. INTRODUCTION

DCT based algorithm such as JPEG international standard [1] is one of the most popular method to compress image data, however it is not suitable when high quality of a decoded image is required. Because it is unavoidable to have distortion due to quantization of DCT coefficients.

A lossless coding method based on a classical DPCM [2] guarantees high quality of a decoded image since it does not contain any quantization, however data compression ratio is not so high.

Towards a new JPEG standard (JPEG-LS) [3], so many lossless algorithms have been proposed. Context modeling and non-linear predictors such as MED in LOCO [4], FIR-Boolian [5], GAP in CALIC [6], etc. are proposed to afford local statistics (edges) in an image signal. However these predictors are not suitable for progressive transmission because of no *anti-aliasing* filter.

Reversible wavelet (R-Wavelet) is inherently advantageous for progressive transmission since it produces a low resolution image as a part of its encoding procedure. So far, various R-Wavelets have been proposed [7,8,9] including S transform [10], SP transform [11], TS transform in CREW [12], lifting structured filter banks [13], etc..

In this report, we propose a new lossless / lossy progressive coding method combining R-Wavelet and lossless multi-channel prediction (LM-Prediction) [14] and also introducing a proper "scaling" before quantization. The method has two modes – "losslesss" mode and "lossy" mode – and also "progressive" functionality. We also propose an optimization procedure of the coding system for an arbitrary input image signal introducing a new criterion – lossless / lossy unified coding gain.

2. OVERVIEW OF THE SYSTEM

Figure 1 illustrates a block diagram of the proposed method. First of all, one dimensional R-Wavelet is applied to an input signal "x" in vertical direction and then in horizontal. As a result, four band signals "y_{LL}", "y_{HL}", "y_{LH}", "y_{HH}" are obtained. An example of the signals are indicated in figure 4. After this, we apply LM-Prediction detailed in ***3.2*** so that still remaining correlation of the signals is fully utilized and the system can be optimized in lossless mode for the input signal "x".

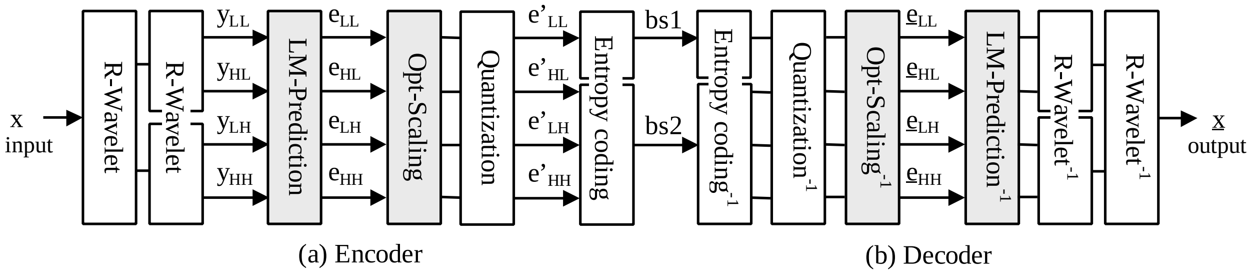


Figure 1. The proposed lossless / lossy progressive coding

Output signals of the LM-Prediction "e_{LL}" and "e_{HL}, e_{LH}, e_{HH}" are separately entropy coded and embedded in two bit streams "bs1" and "bs2" respectively. In case of progressive transmission, only "bs1" is decoded at first to produce a down-scaled version (low frequency component) of "x". The bit stream "bs2" is used to decode the original signal "x" without any loss.

In "lossy mode", "e_{LL}, e_{HL}, e_{LH}, e_{HH}" are quantized. We introduce a proper "scaling" detailed in ***4.1*** before the quantization so that the system can be optimized for the input signal "x" in lossy mode.

3. OPTIMIZATION IN LOSSLESS MODE

3.1 R-Wavelet [10]

In this report, we use S transform [10] as a R-Wavelet. It decomposes an M point input signal x(m), m=0,1,⋯,M-1 into two band signals $y_L(n)$ and $y_H(n)$, n=0,1,⋯,N-1, (N=M/2) by

$$\begin{pmatrix} y_L(n) \\ y_H(n) \end{pmatrix} = \begin{pmatrix} \mathrm{Fl}\left[\dfrac{x(2n)+x(2n+1)}{2}\right] \\ x(2n)-x(2n+1) \end{pmatrix} \tag{1}$$

where Fl[ ] denotes *flooring* into an integer. Applying equation (1) to the original image $x(m_1,m_2)$ in two directions, four band signals $y_{LL}(n_1,n_2)$, $y_{HL}(n_1,n_2)$, $y_{LH}(n_1,n_2)$ and $y_{HH}(n_1,n_2)$ are produced. (see figure 1).

3.2 LM-Prediction [14]

After the S transform, we adopt LM-Prediction [14] in figure 2 to make the best use of remaining correlation of the band signals. Here in after, we denote {LL, HL, LH, HH} as {0, 1, 2, 3} respectively. Then, the band signals $y_p(n_1,n_2)$ are predicted each other by

$$e_p(n_1,n_2) = y_p(n_1,n_2) + \mathrm{R}\left[\sum_{q=0}^{p-1} \mathrm{F}_{qp}[y_q(n_1,n_2)]\right] \qquad p=3,2,1 \tag{2}$$

where R[ ] denotes "*rounding*" into an integer. F_{qp}[ ] denotes non-separable two dimensional FIR filter described with filter coefficients $c_{qp}^{(i,j)}$ as

$$\mathrm{F}_{qp}[y_q(n_1,n_2)] = \sum_j \sum_i c_{qp}^{(i,j)} y_q(n_1+i, n_2+j) \tag{3}$$

We determine coefficients $c_{qp}^{(i,j)}$ under the lossless coding gain defined in ***3.3***.

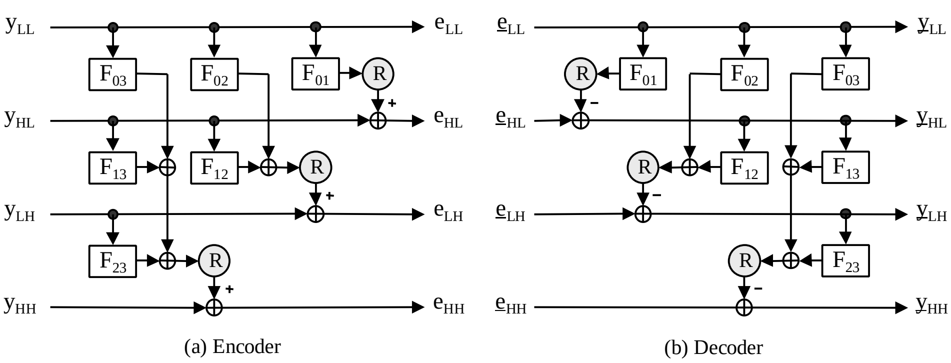


Figure 2. Lossless multi channel prediction (LM-Prediction). F_{pq} and R indicate an FIR filter and a rounding.

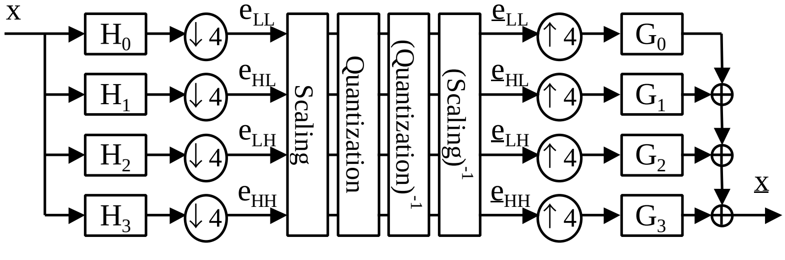


Figure 3. Equivalent expression of figure 1.

3.3 Lossless Coding Gain

In lossless mode, our purpose is to minimize the total bit rate B_{LSL} [bpp]

$$B_{LSL} = \sum_{p=0}^{P-1} w_p B_p, \quad \sum_{p=0}^{P-1} w_p = 1 \tag{4}$$

where B_p denotes bit rate of the signal $e_p(n_1,n_2)$. Since it is well known that a bit rate is estimated by a signal's variance [2]

$$\begin{aligned} B_p &= \log_2 \gamma_p \sqrt{\sigma_{ep}^2}, \quad p = 0,1,\cdots,P-1 \\ B_{PCM} &= \log_2 \gamma \sqrt{\sigma_x^2} \end{aligned} \tag{5}$$

equation (4) becomes

$$B_{LSL} = \log_2 \prod_{p=0}^{P-1} \left(\gamma_p \sqrt{\sigma_{ep}^2}\right)^{w_p} \tag{6}$$

In this report, we define a new criterion *"lossless coding gain"* to evaluate system's performance by

$$G_{LSL} = 20\log_{10} \frac{2^{B_{PCM}}}{2^{B_{LSL}}} = G_{LSL}^* + c_1 \tag{7}$$

Substituting equations (5) and (6) into (7), the lossless coding gain is estimated by

$$\boxed{G_{LSL}^* = 10\log_{10} \frac{\sigma_x^2}{\prod_{p=0}^{P-1} (\sigma_{ep}^2)^{w_p}}} \tag{8}$$

$$c_1 = 20\log_{10} \frac{\gamma}{\prod_{p=0}^{P-1} (\gamma_p)^{w_p}}$$

We determine filter coefficients in equation (3) so that the lossless coding gain in equation (8) becomes maximum. Please refer to [15] for details.

4. OPTIMIZATION IN LOSSY MODE

4.1 Scaling and Quantization

In lossy mode, the signals $e_p(n_1,n_2)$ are quantized, therefore reconstructed image $\underline{x}(m_1,m_2)$ contains distortion. Prior to the quantization, we introduce scaling of $e_p(n_1,n_2)$ by parameters α_p, namely

$$e_p'(n_1,n_2) = R\left[\frac{e_p(n_1,n_2)}{\Delta a_p}\right] \tag{9}$$

where Δ is a step size of the quantization.

Our purpose is to determine a scaling parameter set $\{\alpha_p\}$ which minimizes the variance

$$\sigma_{LSY}^2 = \sum_{n2=0}^{N2-1} \sum_{n1=0}^{N1-1} \{\underline{x}(n_1,n_2) - x(n_1,n_2)\}^2 \tag{10}$$

under a given total bit rate B_{LSY}

$$B_{LSY} = \sum_{p=0}^{P-1} w_p B'_p \quad \text{[bit/pixel]} \tag{11}$$

where B'_p is a bit rate of $e'_p(n_1,n_2)$. This problem is essentially equivalent to the "optimum bit allocation" [2].

4.2 Optimization problem

To simplify the problem above, we use figure 3 which is equivalent to figure 1 when rounding "R" is neglected. Using other relations

$$\sigma_{LSY}^2 = \sum_{p=0}^{P-1} w_n \|G_p\|^2 \cdot \frac{(\Delta a_p)^2}{12} \tag{12}$$

$$B'_p = B_p - \log_2(\Delta a_p) \tag{13}$$

and equation (5) and (11), our purpose is summarized to determine α_p which minimizes

$$\sigma_{LSY}^2 = \frac{2^{-2B_{LSY}}}{12} \cdot \prod_{q=0}^{P-1} (\gamma_q \sqrt{\sigma_{ep}^2})^{2w_q} \cdot \Omega \tag{14}$$

where

$$\boxed{\Omega = \sum_{p=0}^{P-1} \frac{\|G_p\|^2 \cdot a_p^2}{\prod_{q=0}^{P-1} (a_q^2)^{w_q}}} \tag{15}$$

$$\|G_p\|^2 = \sum_{k2} \sum_{k1} g_p^2(k_1,k_2)$$

and $g_p(k_1,k_2)$ is a filter coefficient of G_p in figure 3.

4.3 Lossless / Lossy Unified Coding Gain

Next, we will show relation of the *"lossless"* coding gain in equation (8) and the *"lossy"* coding gain [2] defined by

$$G_{LSY} = 10\log_{10} \frac{\sigma_{PCM}^2}{\sigma_{LSY}^2} \tag{16}$$

σ^2_{PCM} is a variance of quantization error with step size δ where

$$\begin{aligned} \sigma_{PCM}^2 &= \frac{\delta^2}{12} \\ B_{LSY} &= B_{PCM} - \log_2 \delta \end{aligned} \tag{17}$$

Therefore

$$\sigma^2_{PCM} = \frac{1}{12}\left\{\frac{2^{B_{PCM}}}{2^{B_{LSY}}}\right\}^2 \tag{18}$$

Substituting equation (8), (14), (18) into (16), we get

$$G_{LSY} = G^*_{LSL} - 10\log_{10}\Omega + c_1 \tag{19}$$

In this report, we determine the scaling parameters α_p so that Ω is minimized and we use the lossless / lossy unified coding gain

$$\boxed{G^*_{LSY} = G^*_{LSL} - 10\log_{10}\Omega} \tag{20}$$

to evaluate system's performance in lossy mode.

5. SIMULATION

5.1 Optimum LM-Prediction

We confirm effectiveness of the LM-Prediction using an example below.

$$\begin{pmatrix} F_{01}(z_1,z_2) \\ F_{02}(z_1,z_2) \\ F_{13}(z_1,z_2) \\ F_{23}(z_1,z_2) \end{pmatrix} = \begin{pmatrix} c_{01}^{(-1,0)} z_1^{-1} + c_{01}^{(1,0)} z_1 \\ c_{02}^{(0,-1)} z_2^{-1} + c_{02}^{(0,1)} z_2 \\ c_{13}^{(0,-1)} z_2^{-1} + c_{13}^{(0,1)} z_2 \\ c_{23}^{(-1,0)} z_1^{-1} + c_{23}^{(1,0)} z_1 \end{pmatrix} \tag{21}$$

else above

$$F_{qp}(z_1,z_2) = 0, \quad (p,q) \in (0,1,2,3)$$

This example includes only "intrer" band predictions. When we use an "intra" band predictions, we should introduce a local decoding in the encoder. We determined the filter coefficients for each of input images so that the lossless coding gain is maximized using LMS method (see [15]). These values must be embedded into the bit stream as side information.

5.2 Optimum Scaling

We determined the scaling parameters α_p, so that Ω in equation (15) is minimized. As a result, optimum parameters are given by

$$\alpha_p = \sqrt{\frac{\|G_0\|^2}{\|G_p\|^2}}, \quad p \in (0,1,2,3) \tag{22}$$

The filters $G_p(z_1,z_2)$ are expressed with $F_{qp}(z_1,z_2)$ and synthesis transfer functions of the S transform as follows.

$$\begin{pmatrix} G_0(z_1,z_2) \\ G_1(z_1,z_2) \\ G_2(z_1,z_2) \\ G_3(z_1,z_2) \end{pmatrix} = \begin{pmatrix} 1 & E_{01} & E_{02} & E_{03} \\ 0 & 1 & E_{12} & E_{13} \\ 0 & 0 & 1 & E_{23} \\ 0 & 0 & 0 & 1 \end{pmatrix} \begin{pmatrix} T_0(z_1)T_0(z_2) \\ T_1(z_1)T_0(z_2) \\ T_0(z_1)T_1(z_2) \\ T_1(z_1)T_1(z_2) \end{pmatrix} \tag{23}$$

where

$$E_{ij} = \begin{cases} -\sum_{k=i}^{j-1} F_{kj}(z_1^2, z_2^2)\, E_{ik} & , \quad i < j \\ 1 & , \quad i = j \end{cases}$$

$$\begin{pmatrix} T_0(z) \\ T_1(z) \end{pmatrix} = \begin{pmatrix} 1+z \\ (1-z)/2 \end{pmatrix}$$

5.3 Lossless / Lossy Unified Coding Gain

A combination of the S transform and the LM Prediction (ST+LMP) is compared to the S transform alone (ST). In lossless mode, G^*_{LSL} in equation (8) and G^*_{LSY} in equation (20) are used as criteria in lossless mode and in lossy mode respectively. In average, 1.4 [dB] increase of the lossless coding gain was confirmed. The optimum scaling and LM Prediction improved the lossy coding gain by 3.21 [dB].

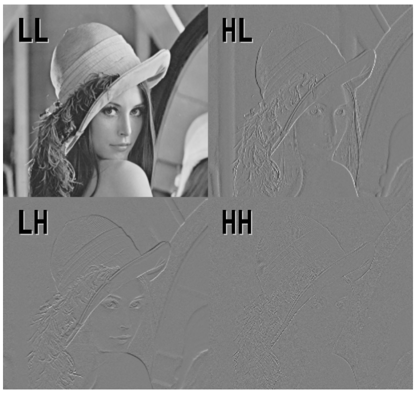
Table 1. Lossless / lossy unified coding gain [dB].

	G^*_{LSL} (lossless)		G^*_{LSY} (lossy)		
	ST	ST +LMP	ST	ST+LMP	
				no scaling	with scaling
Aerial	7.25	8.77	-0.71	0.47	2.57
Barbara	10.03	12.43	2.07	4.16	6.21
Chest X	6.26	6.55	-1.70	-1.63	0.41
Church	5.37	6.69	-2.59	-1.66	0.46
Couple	11.28	13.01	3.32	4.70	6.81
Fruits	10.60	11.00	2.64	2.80	4.86
Girl	12.72	14.76	4.76	6.48	8.57
Lena	14.39	16.73	6.43	8.46	10.55
Moon	9.32	10.15	1.36	1.90	3.98
Average (diff.)	*9.69 (0.0)*	*11.12 (+1.4)*	*1.73 (0.0)*	*2.85 (+1.1)*	*4.94 (+3.21)*

6. SUMMARY

We have proposed a new lossless / lossy progressive coding of digital still images. The method is based on reversible wavelet, lossless multi channel prediction and optimum scaling. We have optimized the system in both of lossless mode and lossy mode introducing the lossless / lossy unified coding gain. We confirmed effectiveness of our proposal by means of the coding gain.

(a) Input image “x”.



(b) Band signals “y_{LL}”, “y_{HL}”, “y_{LH}”, “y_{HH}”

(c) Band signals “e_{LL}”, “e_{HL}”, “e_{LH}”, “e_{HH}”

Figure 4. An image example “Lena”.

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