LOSSLESS, NEAR-LOSSLESS AND LOSSY ADAPTIVE CODING BASED ON THE LOSSLESS DCT*

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ABSTRACT

This report proposes a new coding method, which realizes lossless, near-lossless and lossy coding with a unified single algorithm. The method is based on a lifting-structured lossless DCT, which maps an "integer" input vector to also an "integer" output vector. The proposed method can be used as not only a high performance "*lossless*" encoder but also a "*lossy*" encoder compatible to conventional DCT-based methods such as JPEG and MPEG. In addition, it has a "*near-lossless*" mode in which compression ratio is better than lossless mode remaining quite good quality of decoded images. In "*lossless*" mode, only the lossless DCT and an entropy coder are used. In "*lossy*" mode, transformed coefficients are quantized and then entropy coded in the same manner as a conventional method for compatibility. In "*near-lossless*" mode, signals are quantized before DCT to avoid SNR degradation due to "rounding" operations in the lossless DCT.

1. INTRODUCTION

In the field of digital image coding, the DCT (discrete cosine transform) dominates all over the world as a key module of international standards such as JPEG [1] and MPEG [2]. These algorithms reduce data volume at the compression rate of 1/20 to 1/30 using "quantization" of DCT coefficients. However distortion of decoded images is unavoidable with these "lossy" coding.

For the purpose of use in some fields that require perfect quality of decoded images (e.g. medical engineering, fine art data base), a compaction algorithm without any loss of decoded images is desirable. For this requirement, some "lossless" and "lossy" unified coding is proposed such as JPEG-LS [3] and JPEG-2000 [4,5]. However, these algorithms are based on adaptive prediction and lifting-structured wavelet transforms respectively, they have no compatibility with conventional DCT-based lossy algorithms such as MPEG and JPEG.

In this report, we propose a "lossless", "near-lossless" and "lossy" unified coding algorithm, which has compatibility with the DCT-based "lossy" one. The method is based on the lossless DCT, we have developed [6], which maps an "integer" input vector to also an "integer" output vector. With this method, one can attain not only "lossless" and "near-lossless" coding of images for excellent image quality but also "lossy" coding compatible with conventional MPEG and JPEG algorithms as illustrated in fig. 1.

* A part of the content has been appeared in Ref. [9]

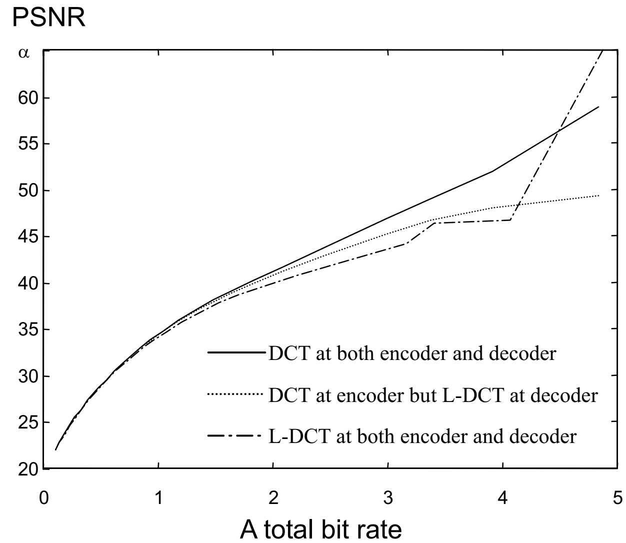
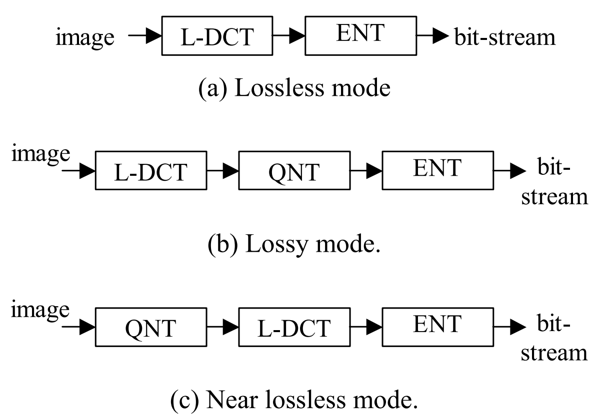


Fig. 1 Rate distortion curve of "Barbara"

2. THE UNIFIED DCT METHOD

In "lossless" mode, only the lossless DCT and an entropy coder are used as explained in **2.1** In "lossy" mode, transformed coefficients are quantized and then entropy coded in the same manner as a conventional method for compatibility as in **2.2**. In "near-lossless" mode, signals are quantized before DCT as in **2.3** to avoid SNR degradation due to "rounding" operations in the lossless DCT.



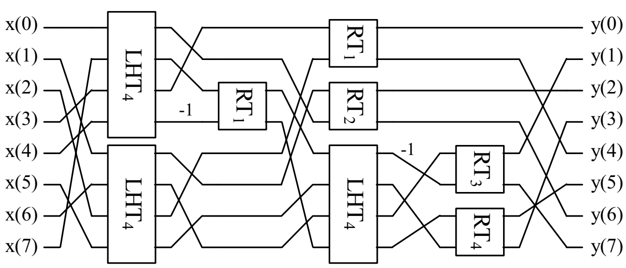
"DCT"=lossy DCT   "L-DCT"=lossless DCT
"ENT"=entropy coder   "QNT"=quantization

Fig.2 Three encoding modes of the proposed method.

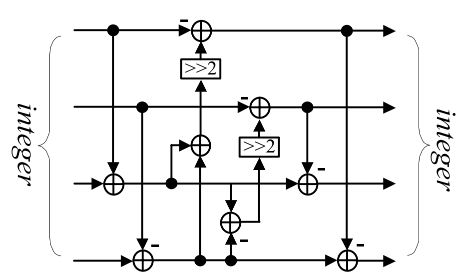
2.1. Lossless Mode

In "lossless" mode, the lossless DCT (L-DCT), we have developed [6], and an entropy coder is used for encoding an image signal as illustrated in fig. 2 (a). The L-DCT maps an integer vector to a vector expressed with also integer. Different from the conventional DCT, one can attain high performance of "lossless" coding of images since it does not increase variance of the signal in spite of using no quantization. The L-DCT is lifting-structured and it contains "rounding" operations as illustrated in fig. 3 where multiplier vectors are given by

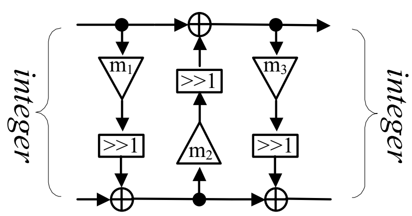
$$
\begin{aligned}
M_1 &= \left[1-\sqrt{2} \quad 1/\sqrt{2} \quad 1-\sqrt{2}\right] \\
M_2 &= \left[\frac{\sin(\pi/8)-1}{\cos(\pi/8)} \quad \cos(\pi/8) \quad \frac{\cos(3\pi/8)-1}{\cos(\pi/8)}\right] \\
M_3 &= \left[\frac{1-\cos(3\pi/16)}{\sin(3\pi/16)} \quad -\sin(3\pi/16) \quad \frac{1-\cos(3\pi/16)}{\sin(3\pi/16)}\right] \\
M_4 &= \left[\frac{\cos(\pi/16)-1}{\sin(\pi/16)} \quad \sin(\pi/16) \quad \frac{\cos(\pi/16)-1}{\sin(\pi/16)}\right]
\end{aligned} \tag{1}
$$



(a) 8 point Lossless DCT (L-DCT) [6]



(b) 4 point Lossless Hadamard Transform (LHT_4) [6]. ">>2" denotes two bit shift to the right ("*rounding*").



(c) 2 point Lossless Rotation (RT_i). M_i=[m_1 m_2 m_3] is the multiplier vector.

Fig.3 L-DCT and its components LHT_4 and RT_i.

2.2. Lossy Mode

In "lossy" mode, as illustrated in fig. 2 (b), quantization is introduced before the DCT in the same manner as the conventional methods. In this mode, image data compressed with JPEG or MPEG such as DVD software can be decoded with the proposed method. The only one difference between conventional and proposed is whether DCT or L-DCT is used. In low and middle bit rate, the "rounding" error due to the rounding operations in the L-DCT is negligible compared to the quantization error. Therefore there is no significant difference between DCT and L-DCT in practice.

2.3. Near-Lossless Mode

On the other hand, in high bit rate, the rounding error emerges when the quantization error becomes small. Therefore, we propose to change the ordering of the quantization and the L-DCT as illustrated in fig. 2 (c) in "near-lossless" mode. As a result, we can avoid SNR degradation due to the "rounding" operations in the L-DCT. In addition, it is guaranteed that the error signal $e(n_1,n_2)$ in the decoded signal $\underline{x}(n_1,n_2)$ defined by

$$e(n_1, n_2) = \underline{x}(n_1, n_2) - x(n_1, n_2) \tag{2}$$

where $x(n_1,n_2)$ denotes the original signal, does not exceed a specified value, namely,

$$\max\{e(n_1, n_2)\} \quad \le \quad \Delta/2 \tag{3}$$

This "near-lossless" condition [7] can not be satisfied in the lossless Wavelets of JPEG-2000 since its synthesis filters modify probability density function (PDF) of the quantization error values to generalized Gaussian like one. JPEG-LS can realize the near-lossless coding with introducing the local decoding into the prediction based filtering, however, it has no compatibility with the conventional DCT-based algorithms.

3. TURNING POINT DETERMINATION

As a conclusion, the proposed method uses the L-DCT, quantization and an entropy coder (i.e. arithmetic coding) switching ordering of them as in the fig. 2 according to coding modes. The turning point of the "lossy" mode and the "near-lossless" mode is considered in this section.

3.1. Experimental approach

Rate distortion curves for each of three methods as below:

Method (i)	DCT + QNT + ENT
Method (ii)	L-DCT + QNT + ENT
Method (iii)	QNT + L-DCT + ENT

are investigated for the image "Lena" in fig. 4. Method (i) is the conventional DCT-based algorithm labeled as "DCT-QNT" in the figure. This is the best in PSNR (peak signal to noise ratio) among three, however it cannot attain lossless coding indicated as infinite value of PSNR. In method (ii) labeled as "LDCT+QNT", the conventional DCT is replaced by the L-DCT in fig. 3. This can realize high performance of lossless coding of images, however SNR decreases in high bit rate due to the "rounding" operations in the L-DCT. On the other hand, the method (iii) labeled as "QNT + LDCT", which is the exchanged version of L-DCT and quantization of the method (ii), does not decrease SNR in high bit rate since there is no rounding error in the decoded image. Therefore we propose the following method.

1. In low and middle bit rate, use fig. 2 (b) as a lossy coding compatible with the conventional DCT-based methods.
2. In high bit rate, use fig. 2 (c) as a near-lossless coding which satisfies the condition in eq. (3).
3. In lossless mode, use fig. 2 (a) as a high performance lossless coding.

The turning point of 1. and 2. above is experimentally evaluated and result is summarized in table 1. As indicated in table 1, the turning point exists at around 6 of the quantization step size. Notice that it varies from image to image and uniform step size is used in this experiment.

3.2. Theoretical support [9]

In this section, we analyze a theoretical support of each of the three rate-distortion curves in fig. 4 under the following assumptions: 1) All filters are linear and time invariant. 2) Correlations between each of errors from rounding operations and the signals are zero. (statistically independent) 3) Power spectrums of errors are flat. The PSNR in this report is defined from

$$PSNR = 10\log_{10}\left(\frac{255^2}{\sigma_E^2}\right) \tag{4}$$

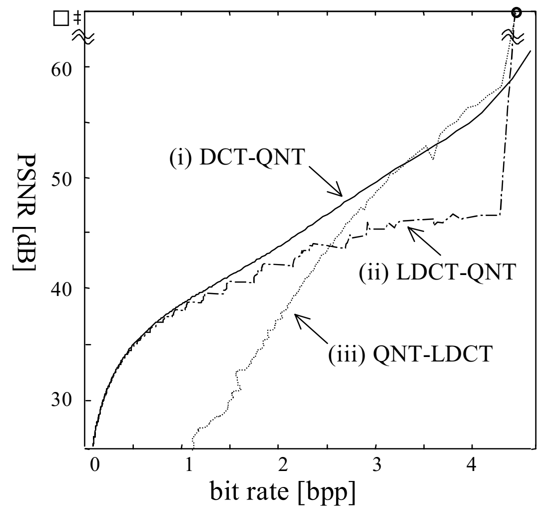
where σ_E^2 denotes a variance of total errors between input signals and decoded signals. A total bit rate B is given by taking average of the bit rate of each band B_i as

$$B = \frac{1}{8}\sum_{i=0}^{7} B_i \tag{5}$$

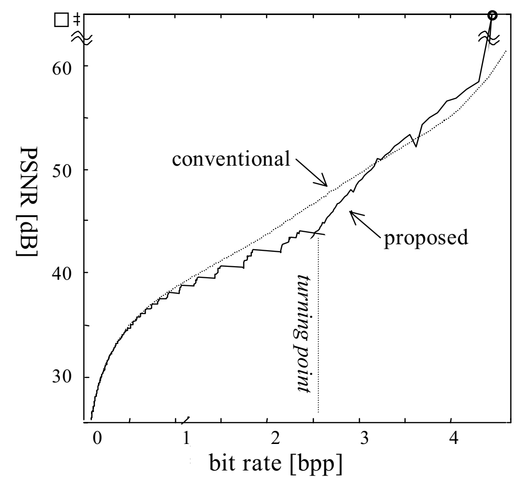
where the bit rate of each DCT band is related to its variance (sigma) as

$$B_i = \log_2 \gamma_i \sqrt{\frac{\sigma_i^2}{\Delta^2}} \tag{6}$$

where "Δ" is a step size of quantization. "γ_i" and "σ^2_i" denotes a constant value determined by PDF of the i^{th} band signals and variance of transformed signal of the i^{th} band signals, respectively. The total error contained in the decoded signal is a sum of the quantization error (σ_Q^2) and the rounding error (σ_R^2):



(a) Comparison of three methods.



(b) Proposed method.

Fig.4 Rate distortion curves (Lena).

Table 1 Experimental result of the turning point. The standard images are 256*256 pixel^2 and 8-bit gray color.

Name of the standard images	Lena	Baboon	Church	Couple	Average
Step size	6.125	8.125	7.25	4.125	6.4

$$\sigma_N^2 = \sigma_Q^2 + \sigma_R^2 \tag{7}$$

where variance of quantization is calculated from

$$\sigma_Q^2 = \frac{\Delta^2}{12} \tag{8}$$

From the previous assumptions, variances of rounding errors in each band are

$$\begin{bmatrix}\sigma_{N_{R0}}^2 & \sigma_{N_{R1}}^2 & \sigma_{N_{R2}}^2 & \sigma_{N_{R3}}^2 & \sigma_{N_{R4}}^2 & \sigma_{N_{R5}}^2 & \sigma_{N_{R6}}^2 & \sigma_{N_{R7}}^2\end{bmatrix} = \begin{bmatrix}0.2083 & 0.2758 & 0.2378 & 0.3308 & 0.2226 & 0.3925 & 0.2161 & 0.44\end{bmatrix} \tag{9}$$

In method (ii), we can write a total bit rate in term of quantization step size as

$$B_{T,(ii)} = \log_2 \prod_{i=0}^{7} (\gamma_i)^{1/8} + \log_2 \prod_{i=0}^{7} \left(\sqrt{\sigma_{XH_i}^2} \right)^{1/8} - \log_2 \sqrt{\Delta_{(ii)}^2} \quad (10)$$

where $\sigma_{XH_i}^2$ denotes variance of transformed signal in i[th] subband. A relation between PSNR and quantization step size is

$$PSNR_{(ii)} = 58.92 - 10 * \log_{10} (\Delta_{(ii)}^2 + 6.96) \quad (11)$$

From eq. (10) and eq. (11), we can find relation between a total bit rate and PSNR as

$$PSNR_{(ii)} = 58.92 - 10 * \log_{10} \left(\frac{\prod_{i=0}^{7} \left(\gamma_i \sqrt{\sigma_{XH_i}^2} \right)^{1/8}}{2^{2B_{T,(ii)}}} + 6.96 \right) \quad (12)$$

Eq. (12) confirms that slope in method (ii) is less than 6.02 dB/bits. Similarly, we can write a total bit rate in method (iii) in term of quantization step size as

$$B_{T,(iii)} = \log_2 \prod_{i=0}^{7} (\gamma_i)^{1/8} + \log_2 \prod_{i=0}^{7} \left(\sqrt{\sigma_{XH_i}^2 + \Delta_{(iii)}^2 \sigma_{NR_i}^2} \right)^{1/8} - \log_2 \sqrt{\Delta_{(iii)}^2} \quad (13)$$

and a relation between PSNR and quantization step size is

$$PSNR_{(iii)} = 58.92 - 10 * \log_{10} \Delta_{(iii)}^2 \quad (14)$$

From eq. (13) and eq. (14), we can find relation between a total bit rate and PSNR as

$$PSNR_{(iii)} = 58.92 - 10 * \log_{10} \prod_{i=0}^{7} \frac{\left(\gamma_i \sqrt{\sigma_{XH_i}^2 + \Delta_{(iii)}^2 \sigma_{NR_i}^2} \right)^{1/8}}{2^{2B_{T,(iii)}}} \quad (15)$$

Eq. (15) confirms that slope in method (iii) is more than 6.02 dB/bit. At turning point, a total bit rate and PSNR of method (ii) are the same as those of method (iii) as

$$PSNR_{(ii)} = PSNR_{(iii)} \quad and \quad B_{T,(ii)} = B_{T,(iii)} \quad (16)$$

From condition of the turning point in eq. (16), if we know variance of transformed signals, we can calculate quantization step size by using eq. (11) – eq. (16). We apply 1D AR(1) model as input signals to confirm accuracy this theoretical support as shown in table 2. Notice that details of theoretical support are in Ref. [9]

Table 2 The value of quantization step size $\Delta_{(ii)}$ at the turning point based on 1D AR(1) model with given ρ.

ρ	0.99	0.95	0.9	0.85	0.8
$\Delta_{(ii)}$ (Practical)	2.8	7.9	9.0	10.0	11.0
$\Delta_{(ii)}$ (Theoretical)	2.71	6.89	8.14	8.61	9.36
Difference (%)	3.2	12.8	9.6	13.9	14.9

4. CONCLUSION

We have proposed a new unified single algorithm, which can encode image signal in "lossless", "near-lossless" and "lossy" mode. The method is based on the L-DCT which maps an "integer" input vector to also an "integer" and is based on lifting-structure and rounding operations. The proposed method can be used as a "lossless" encoder and also a "lossy" encoder compatible to conventional JPEG and MPEG. The "near-lossless" coding is also possible in which compression ratio is better than lossless mode remaining quite good picture.

In "lossless" mode, quantization is not used. In "lossy" mode, output signal of the L-DCT is quantized. In "near-lossless" mode, signal is quantized before the L-DCT to avoid SNR degradation due to "rounding" error. Turning point of the "lossy mode" and "near lossless mode" is investigated.

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