Lossless/lossy Image Compression based on Non-Separable Two-Dimensional L-SSKF

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Abstract

In this report, we propose a non-separable two-dimensional (2D) Lossless Symmetric Short Kernel Filter (L-SSKF) for image compression. Filter characteristics of our proposed L-SSKF are the same as those of conventional 2D L-SSKF based on applying 1D L-SSKF twice but our coding performance is better due to reduction of rounding effects. Simulation results confirm effectiveness of our proposed L-SSKF.

Key words: L-SSKF, non-separable, two-dimensional, filter bank, rounding effect, image compression

1. Introduction

Many researchers have been paying attention to lossless algorithm to serve some applications that require a high-quality decoded image such as medical image. The Lossless Symmetric Short Kernel Filter (L-SSKF) [1] is one of the famous lossless algorithms because the L-SSKF-based coding system can provide not only lossy coding but also lossless coding thanks to lifting structures (LS) and rounding operations. However, the error generated from rounding operation [2] causes PSNR degradation in lossy coding when quantization is applied. The conventional L-SSKF is a one-dimensional (1D) filter bank (FB) constructed from double LS. To perform 2D FB for image application, the 1D L-SSKF is applied twice in horizontal and vertical dimension, successively.

In this report, we propose a non-separable 2D L-SSKF. Number of rounding operations of our proposed L-SSKF is less than that of conventional 2D L-SSKF, whereas filter characteristics of our proposed L-SSKF are the same as those of conventional 2D L-SSKF when the error generated from rounding operation is negligible. Therefore, coding performance of our proposed L-SSKF is better than that of the conventional 2D L-SSKF in lossy coding, especially at high bit rate when quantization errors are relatively small compared to rounding errors. Moreover, coding performance of our proposed L-SSKF is slightly better than that of the conventional 2D L-SSKF in lossless coding.

This report is organized as follows. We review the conventional 2D L-SSKF in section 2. Then, we propose non-separable 2D L-SSKF for image compression in section 3. Simulation results confirm effectiveness of our proposed L-SSKF in both lossless coding and lossy coding in section 4. Finally, we summarize our proposed L-SSKF in section 5.

2. The Conventional Two-Dimensional (2D) L-SSKF

2.1 Signal processing of the conventional 2D L-SSKF

We construct the conventional 2D L-SSKF by applying conventional 1D L-SSKF in horizontal and vertical dimension as illustrated in fig. 1. Input signals (X) are decomposed into 4 subbands (Y_{LL}, Y_{LH}, Y_{HL}, Y_{HH}). For example, Y_{LH} indicates horizontally low-passed and vertically high-passed subband. The z_1 and z_2 denotes horizontal and vertical dimension, respectively. The Q_{LL}, Q_{LH}, Q_{HL}, Q_{HH} denote quantization in subband LL, LH, HL, and HH, respectively. The LS denotes lifting structure. The ® and "↓2" denote a rounding operation and a downsampler by two [3]. As shown in fig. 1, six rounding operations are required to perform conventional 2D L-SSKF.

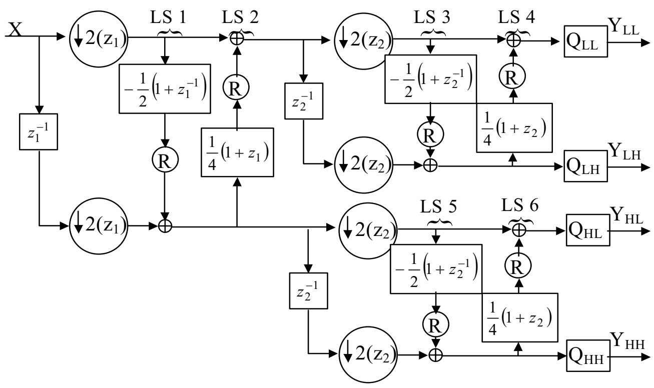

Fig. 1 Signal processing of the conventional 2D L-SSKF

2.2 An equivalent expression of the conventional 2D L-SSKF

We analyze all signal processing in this report based on the following assumptions. (1) All filters in this report are linear and time-invariant filters, and (2) Correlations between each of the errors and the signals are zero (statistical independence). In this report, we use z-transform expression defined by

$$X(z) = \sum_{k=0}^{K-1} x(k)\, z^{-k} \tag{1}$$

where x(k) denotes signal's intensity. Value of x(k) for image signal is given in "integer". Signal processing in fig. 1 can be replaced by its equivalent expression in fig. 2 where its filter characteristics are

$$\begin{bmatrix} H_{LL,C} \\ H_{LH,C} \\ H_{HL,C} \\ H_{HH,C} \end{bmatrix} = T_{C4} * T_{C3} * T_{C2} * T_{C1} \begin{bmatrix} 1 \\ z_1^{-1} \\ z_2^{-1} \\ z_1^{-1} z_2^{-1} \end{bmatrix} \tag{2}$$

where transform matrix (T_{ci}) are based on parameters of the conventional 2D L-SSKF in fig. 1 as

$$T_{C1}=\begin{bmatrix}1 & 0 & 0 & 0\\ 0 & 1 & 0 & 0\\ -\frac{1}{2}(1+z_1^{-1}) & 0 & 1 & 0\\ 0 & -\frac{1}{2}(1+z_1^{-1}) & 0 & 1\end{bmatrix} \quad (3)$$

$$T_{C2}=\begin{bmatrix}1 & 0 & \frac{1}{4}(1+z_1) & 0\\ 0 & 1 & 0 & \frac{1}{4}(1+z_1)\\ 0 & 0 & 1 & 0\\ 0 & 0 & 0 & 1\end{bmatrix} \quad (4)$$

$$T_{C3}=\begin{bmatrix}1 & 0 & 0 & 0\\ -\frac{1}{2}(1+z_2^{-1}) & 1 & 0 & 0\\ 0 & 0 & 1 & 0\\ 0 & 0 & -\frac{1}{2}(1+z_2^{-1}) & 1\end{bmatrix} \quad (5)$$

$$T_{C4}=\begin{bmatrix}1 & \frac{1}{4}(1+z_2) & 0 & 0\\ 0 & 1 & 0 & 0\\ 0 & 0 & 1 & \frac{1}{4}(1+z_2)\\ 0 & 0 & 0 & 1\end{bmatrix} \quad (6)$$

According to Reichel's report [2], non-linearity of rounding operation generates additive noise and then the noise is propagated through FB to the reconstructed image. Therefore, rounding errors in its equivalent expression are

$$\begin{bmatrix}R_{LL,C}\\ R_{LH,C}\\ R_{HL,C}\\ R_{HH,C}\end{bmatrix}=\begin{bmatrix}R_4\\ 0\\ R_6\\ 0\end{bmatrix}+T_{C4}\begin{bmatrix}0\\ R_3\\ 0\\ R_5\end{bmatrix}+T_{C4}*T_{C3}\begin{bmatrix}R_2\\ R_2\\ 0\\ 0\end{bmatrix}+T_{C4}*T_{C3}*T_{C2}\begin{bmatrix}0\\ 0\\ R_1\\ R_1\end{bmatrix} \quad (7)$$

where rounding errors R_i denotes additive errors generated from rounding operation in lifting structure i^{th}.

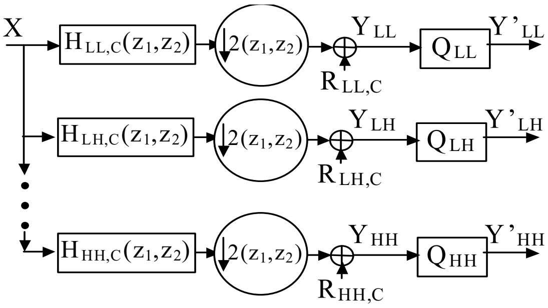

Fig. 2 An equivalent expression of the conventional 2D L-SSKF

3. Our Proposed Non-Separable 2D L-SSKF

3.1 Signal processing of non-separable 2D L-SSKF

In this section, we propose the non-separable 2D L-SSKF based on our objective to reduce rounding effects. Filter characteristics of our proposed L-SSKF are the same as those of conventional 2D L-SSKF but our proposed 2D L-SSKF requires only four rounding operations as shown in fig. 3. Because of advantages of non-separable 2D FB, parameters in different LS of conventional 2D L-SSKF can be combined. For example, parameters of LS 1' in fig. 3 are combined from parameters of LS 1 and LS 5 in fig. 1. Therefore, number of rounding operation required to perform 2D L-SSKF is reduced.

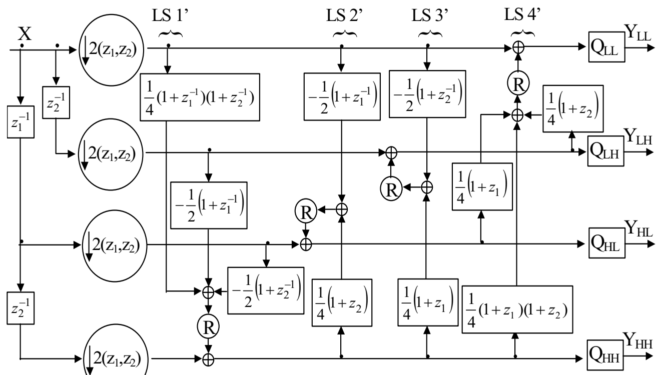

Fig. 3 Signal processing of proposed non-separable 2D L-SSKF

3.2 An equivalent expression of proposed non-separable 2D L-SSKF

Similarly, signal processing in fig. 3 can be replaced by its equivalent expression in fig. 4 where its filter characteristics are

$$\begin{bmatrix}H_{LL,P}\\ H_{LH,P}\\ H_{HL,P}\\ H_{HH,P}\end{bmatrix}=T_{P4}*T_{P3}*T_{P2}*T_{P1}\begin{bmatrix}1\\ z_1^{-1}\\ z_2^{-1}\\ z_1^{-1}z_2^{-1}\end{bmatrix} \quad (8)$$

where transform matrix (T_{pi}) are based on parameters of non-separable 2D L-SSKF in fig. 3 as

$$T_{P1}=\begin{bmatrix}1 & \frac{1}{4}(1+z_2) & \frac{1}{4}(1+z_1) & \frac{1}{4}(1+z_1)(1+z_2)\\ 0 & 1 & 0 & 0\\ 0 & 0 & 1 & 0\\ 0 & 0 & 0 & 1\end{bmatrix} \quad (9)$$

$$T_{P2}=\begin{bmatrix}1 & 0 & 0 & 0\\ -\frac{1}{2}(1+z_2^{-1}) & 1 & 0 & \frac{1}{4}(1+z_1)\\ 0 & 0 & 1 & 0\\ 0 & 0 & 0 & 1\end{bmatrix} \quad (10)$$

$$T_{P3} = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ -\frac{1}{2}(1+z_1^{-1}) & 0 & 1 & \frac{1}{4}(1+z_2) \\ 0 & 0 & 0 & 1 \end{bmatrix} \quad (11)$$

$$T_{P4} = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ \frac{1}{4}(1+z_1^{-1})(1+z_2^{-1}) & -\frac{1}{2}(1+z_1^{-1}) & -\frac{1}{2}(1+z_2^{-1}) & 1 \end{bmatrix} \quad (12)$$

Similarly, rounding errors in its equivalent expression are

$$\begin{bmatrix} R_{LL,P} \\ R_{LH,P} \\ R_{HL,P} \\ R_{HH,P} \end{bmatrix} = \begin{bmatrix} R'_4 \\ 0 \\ 0 \\ 0 \end{bmatrix} + T_{P4} \begin{bmatrix} 0 \\ R'_3 \\ 0 \\ 0 \end{bmatrix} + T_{P4} * T_{P3} \begin{bmatrix} 0 \\ 0 \\ R'_2 \\ 0 \end{bmatrix} + T_{P4} * T_{P3} * T_{P2} \begin{bmatrix} 0 \\ 0 \\ 0 \\ R'_1 \end{bmatrix} \quad (13)$$

where rounding errors R'_i denotes errors generated from rounding operation in lifting structure i^{th}. Eq. (2) - eq. (13) confirm that filter characteristics of our proposed non-separable 2D L-SSKF are the same as those of the conventional 2D L-SSKF but rounding errors of our proposed non-separable 2D L-SSKF are less. Simulation results confirm effectiveness of our proposed 2D L-SSKF in next section.

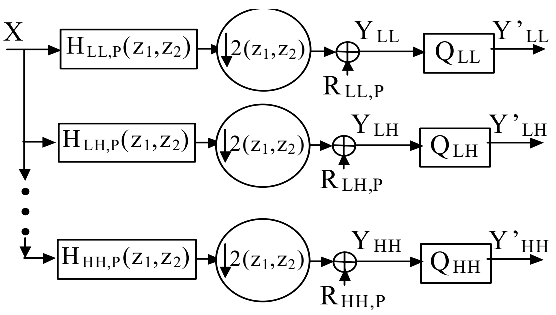

Fig. 4 An equivalent expression of our proposed non-separable 2D L-SSKF

4. Simulation results

In this section, we apply some standard images as input signals to illustrate effectiveness of our proposed method. We illustrate effectiveness of proposed method in lossless coding and lossy coding in section 4.1 and section 4.2, respectively.

4.1 Effectiveness of proposed method in lossless coding

Table 1 compares lossless coding performance of both L-SSKF in term of the entropy rate calculated from

$$H = -\sum_s P_s \log_2 P_s \quad (14)$$

where P_s indicates probability of a symbol "s". The "proposed" and "conventional" indicate "proposed non-separable 2D L-SSKF" and "the conventional 2D L-SSKF", respectively. From results in table 1, our first order entropy rates are slightly less than those of conventional 2D L-SSKF.

Table 1 The entropy rate in lossless coding

Image name	Conventional	Proposed	Improvement (Conventional-Proposed)
Couple	4.762	4.753	0.009
Aerial	5.946	5.946	0
Girl	5.145	5.142	0.003
Flower	5.775	5.767	0.008
Moon	5.354	5.354	0
Barbara	5.530	5.529	0.001
Cartoon	6.299	6.293	0.006
Average	5.544	5.540	0.004

4.2 Effectiveness of proposed method in lossy coding

Table 2 and 3 compare lossy coding performance of both methods in term of PSNR (Peak Signal to Noise Ratio) defined as

$$PSNR = 10\log_{10}\left(\frac{255^2}{MSE}\right) \quad [dB] \quad (15)$$

where MSE denotes the mean square-error between the original image and the decoded image. From results in table 2 and 3, PSNR of our L-SSKF are higher than those of conventional 2D L-SSKF, especially in high bit rate when quantization errors are small. Fig. 5 and fig. 6 confirm effectiveness of our proposed method in term of rate distortion curves of "Barbara" and "Girl", respectively.

Table 2 PSNR at total bit rate = 4 bpp

Image name	Conventional	Proposed	Improvement (Proposed- Conventional)
Couple	48.84	49.18	0.34
Aerial	44.76	45.12	0.36
Girl	47.62	48.14	0.52
Flower	46.63	46.99	0.36
Moon	45.79	46.1	0.31
Barbara	42.97	43.23	0.26
Cartoon	45.13	45.51	0.38

Table 3 PSNR at total bit rate = 3 bpp

Image name	Conventional	Proposed	Improvement (Proposed- Conventional)
Couple	45.41	45.74	0.33
Aerial	39.65	39.80	0.15
Girl	43.97	44.24	0.27
Flower	42.83	43.01	0.18
Moon	41.84	42.04	0.2
Barbara	37.64	37.73	0.09
Cartoon	40.53	40.72	0.19

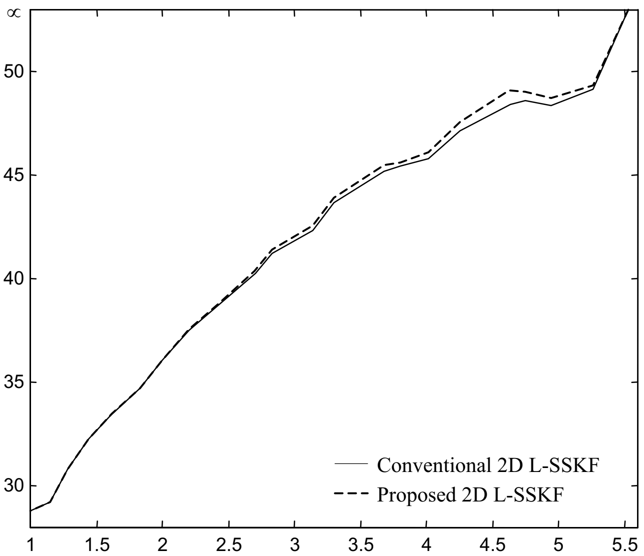

Fig. 5 Rate distortion curve of "Barbara"

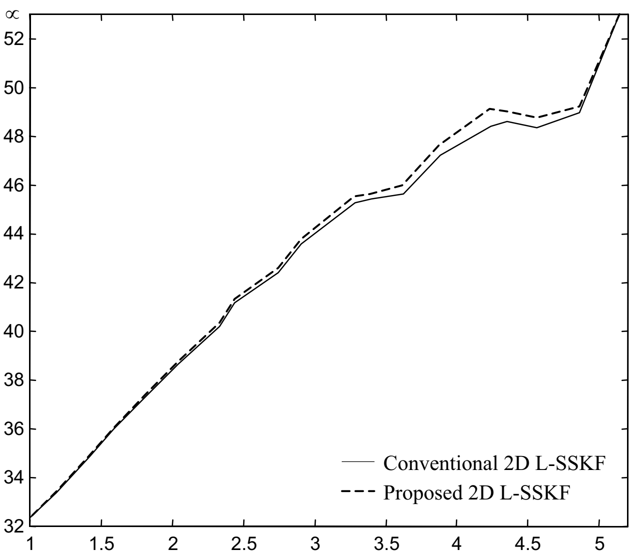

Fig. 6 Rate distortion curve of "Girl"

4.3 Future work

In this report, we confirm effectiveness of our proposed non-separable 2D L-SSKF in both lossless coding and lossy coding based on signal processing in fig. 1 and fig. 3, however our proposed non-separable 2D L-SSKF can be applied to octave decomposition too. Moreover, concept of the non-separable 2D L-SSKF can be applied to reduce the number of rounding operations of other kinds of lossless/lossy wavelets.

4. Conclusion

In this report, we proposed a non-separable 2D L-SSKF with fewer rounding operations. The coding performance of our proposed method is better than that of the conventional 2D L-SSKF because filter characteristics of both method are the same but our proposed L-SSKF has less number of rounding operations required to perform 2D FB. Simulation results confirm effectiveness of our proposed method in both lossless coding and lossy coding. The concept of the non-separable 2D FB is useful to reduce the number of rounding operations to perform 2D FB.

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