Reprinted from PROCEEDINGS OF THE 1992 IEEE
INTERNATIONAL SYMPOSIUM ON CIRCUITS AND
SYSTEMS, San Diego, California, May 10-13, 1992

SUBBAND CODING OF IMAGES WITH CIRCULAR CONVOLUTION

Masahiro Iwahashi[1], Hitoshi Kiya[2], Kiyoshi Nishikawa[2]

[1] Electronics R&D Laboratories,
Nippon Steel Corporation
5-10-1 Fuchinobe, Sagamihara City,
Kanagawa 229, Japan

[2] Faculty of Technology,
Tokyo Metropolitan University
1-1 Minami-Osawa, Hachioji City,
Tokyo 192-03, Japan

Abstract - Herein we introduce a new filtering technique for subband coding (SBC) of images. The method avoids data increase, reduces reconstructed image data distortion, and activates any type of linear phase finite impulse response (FIR) filter. Our procedure involves (1) special methods that generate periodic data from original non-periodic data: (2) identifying relation between periodic data types and filter types: and (3) the selection of appropriate method of rate conversion.

1. INTRODUCTION

Recently, the SBC technique [1-4] has attracted a great deal of attention in the area of image coding. In SBC, an input signal is decomposed into narrow band signals. Each band signal is then encoded appropriately according to its own statistical characteristic. SBC is expected to have useful applications in areas such as parallel processing, layered coding and packet networks. In an SBC system, all filtering procedures should not increase data volume. However, in the case of linear convolution, the filtering procedures expand the data volume.

To avoid this problem, Smith, et al. [5-7] converted an input sequence into a periodic sequence and then utilized circular convolution instead of linear convolution. However, thier techniques are restricted by available filter types and by the filter bank structure. Therefore, we modified their methods to remove these restrictions by considering the properties of circular convolution and rate conversion[10]. In this paper, we will explain the modified filtering procedure in detail and show its effects on two-band decomposition.

2. SUBBAND CODING SYSTEM

The SBC system (see Fig. 1) is composed of an analysis unit, a coding-decoding unit and a synthesis unit. First, the analysis unit decomposes an input signal into two half band signals through H_0 and H_1. Each band signal is then down-sampled by 2:1 down-samplers and fed into the coding-decoding unit. In this unit, the input data is first encoded, then either transmitted or stored and finally decoded. Next, in the synthesis unit, the data is up-sampled by 1:2 up-samplers and filtered through G_0 and G_1 in order to reject any unnecessary band signals. Finally, the two band signals are added together and the final output signal is obtained.

For simplification purposes, we only prove the benefits of our method in one dimension, but this method can be available in two-dimensions as seen by the simulation results.

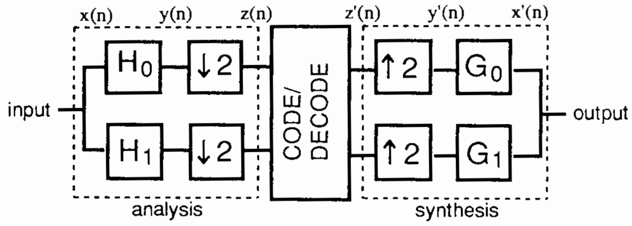


Fig.1 A structure of two-band filter bank. H 1 and G1 are highpass filters and H0 and G0 are lowpass filters.

3. PROBLEMS

In this section, we describe several problems associated with filtering procedures.

3.1 Data Increase

The data increase problem is caused by the filtering procedures used in the analysis unit and in the synthesis unit. In linear convolution, the input data x(n) is convolved with an N-length impulse response h(p). The output y(n) is realized by the following equation:

$$y(n) = \sum_{p=0}^{N-1} h(p)\, x(n-p), \qquad n = 0,1,\dots,L-1. \tag{1}$$

The number of samples of the output data is L+N-1, where L denotes the number of samples of the input data and N denotes the number of terms in the filter's impulse response. This means that by using linear convolution the size of data to be encoded will be increased.

To avoid this problem, Smith, et al. proposed a filtering method using circular convolution instead of linear convolution [5-7]. In general, the input data obtained by scanning image data is non-periodic. Consequently, the input data x(n) should be converted into a periodic sequence $\underline{x}(m)$ before it is convolved (Fig.4). In the Smith's method, the original, non-periodic data which has L sample points (see Fig.2(a)) is transformed into a periodic data sequence $\underline{x}(m)$ with period M (see Fig. 2(b) or Fig. 2(c)). Next, the periodic data is operated on circular convolution. The resulting data is expressed by the following formula:

$$\underline{y}(m) = \sum_{p=0}^{N-1} h(p)\, \underline{x}(m-p), \qquad m = 0,1,\dots,M-1, \tag{2}$$

0-7803-0593-0/92 $3.00 1992 IEEE

where h(p) (p=0,1,2,...,N-1) denotes the impulse response of any arbitrary linear phase FIR filter. Note that the period for both the output y(m) and the input x(m) is M. Therefore, the number of independent data points, L, for the output sequence is the same as the number of data points for the input sequence. Thus, we avoid the data increase problem.

3.2 Data Distortion

In the reports by Smith, et al., two types of periodic data, Fig.2(b) and Fig.2(c), are compared in the respect of data distortion. The distortion is caused by quantization in the coding procedure. As a result of experiments, the method based on symmetric case-A data in Fig.2(c) has more reduced data distortion than that of asymmetric data in Fig.2(b). They concluded that we can reduce the data distortion problem by using symmetric data sequences.

3.3 Filter Types

Currently, linear phase FIR filters are only available with odd order in the Quadrature Mirror Filter (QMF) banks[8]. Also, filters with even order (see Fig.3(b), (d)), such as the Symmetric Short Kernel Filter (SSKF)[3] or perfect reconstruction filters [11], cannot be used with Smith's technique (see Fig.3(a),(c)). The FIR filters which do use Smith's method are restricted to filters of odd order.

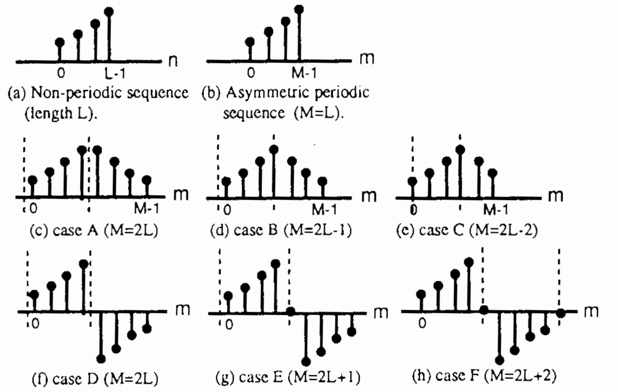


Fig.2 Various methods to generate periodic sequences.
(b)(c) Proposed by Smith et al, (c)-(h) Our proposed methods.
(Broken line indicates a center of symmetry in a period.)

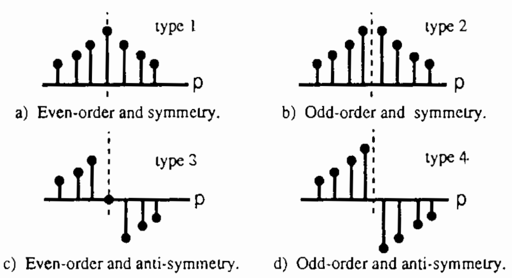


Fig.3 Summary of types of linear phase FIR filters.

4 PROPOSED METHOD

In this section, we will describe our filtering method which can activate any even or odd order linear phase FIR filter.

4.1 Periodic data and Circular Convolution

We shall introduce several new generating methods of periodic data as seen in Fig.2(c)-(h). Our method not only includes Smith's method (see Fig.2(b),(c)), but also introduces five other types of sequences (see Fig.2 (d-h)). All of them have symmetry or anti-symmetry in one period so that data distortion is reduced. These periodic sequences are filtered with one of four types of linear phase FIR filters by circular convolution. The filtered data y(m) is categorized into six types (see Fig.2 (c-h)) in the same way as the input periodic data x(m). As a result of convolution, types of the output data are summarized in table 1. We can see from Table 1 that Smith's method has been limited to case-2 or case-4 filters and case-A input data.

4.2 Down-Sampling

In the analysis unit, the filtered data y(m) is down-sampled to get z(m) (see Fig.4.). This procedure is done under the following conditions:

1) y(m) has a period M and
z(m) has a period M/2; and
2) the down-sampled data z(m) has either
symmetry or anti-symmetry.

If the preceding conditions are met, then the number of independent data points in the down-sampled data z(m) is reduced. Figure 5(a) shows an example for down-sampling case A sequence. In this case, the data does not satisfy condition 2. Only if y(m) is case C or case F, then will z(m) satisfy both of the above conditions. Therefore, the output of circular convolution must be case C or case F (see Table 2). In the down-sampling procedure, there exist two kinds of down-sampling timings. One of them (type-I) maintains a line of symmetry in a period while the other (type-II) does not (see Fig.6). Table-3 shows the effect of these two types of down-sampling on the case C and F.

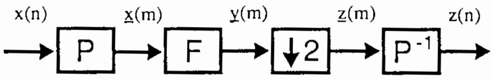


Fig.4 Decimation procedure in analysis unit.

P transformes non-periodic data x(n) to periodic data x (m).
F convolves x(m) with impulse response h(p).
↓2 decimates y(m) to make downsampled data z(m).
P^{-1} transformes periodic data z(m) to non-periodic data z(n) .

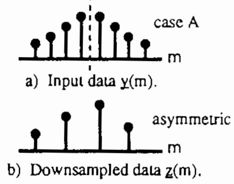


Fig.5 Down-sampling of case A.

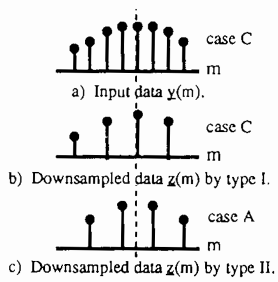


Fig.6 Two kinds of down-sampling of case C.

4.3 Example

The input data x(n) with length L (assuming L is odd) is filtered by a type-1 filter (see Fig.3(a)). As mentioned previously, this filter will not work with Smith's method, but it will work with our method. In the analysis unit, case-C data $\underline{x}(m)$ with period M=2L-2 is formed from the non-periodic input data x(n) and filtered by circular convolution. After convolution with type-1 filters, case-C data $\underline{y}(m)$ with period M is obtained (see Table 2). If M/2 is even, then type-C data $\underline{z}(m)$ with perod M/2=L-1 is obtained by type-I down-sampling (see Table-3). It will then be converted into non-periodic data z(n). If one wishes to filter the low or/and high band signale further, in the case of "tree-structure", for example, then one can pass it through another pair of bandpass filters. This process can be repeated until the desired frequency response is obtained. Afterward, the data z(n) is fed into the next analysis unit, the coding-decoding unit where the data is compress and then either transmitted or stored. The data is then decoded and fed into the synthesis unit. Here the non-periodic data z'(n) is converted to periodic data. It is then up-sampled by interpolating a new data point at the midpoint between each of the original data points. Each new data point has a value of zero. Types of the up-sampled data are summarized in Table-4. Next, the up-sampled data is filtered through the synthesis filters and reformed into non-periodic original data according to Table-1. Finally, the output data is obtained by adding the band signals together.

5. SIMULATION RESULTS

We used symmetric short kernel filters (SSKF) [3] to examine the effects of our method on image data. These case-1 filters are the following:

$$H_0(z) = 1/8(-1+2z^{-1}+6z^{-2}+2z^{-3}-z^{-4})$$

$$H_1(z) = 1/2(1-2z^{-1}+z^{-2})$$

$$G_0(z) = H_1(-z)$$

$$G_1(z) = -H_0(-z) .$$

In this example, the input image data was divided into four bands by the following separable two-dimensional filters:

$$H_{00}(z_1, z_2) = H_0(z_1)\, H_0(z_2)$$

$$H_{01}(z_1, z_2) = H_0(z_1)\, H_1(z_2)$$

$$H_{10}(z_1, z_2) = H_1(z_1)\, H_0(z_2)$$

$$H_{11}(z_1, z_2) = H_1(z_1)\, H_1(z_2) .$$

After decomposition, each band is quantized by means of SBC procedures. In this example, we only use the lowest band signal. Finally, the decomposed data is synthesized by means of separable two-dimensional filters. These filters are composed of $G_0(z)$ and $G_1(z)$ in the same manner as the analysis filters.

A comparison between the method based on symmetric data (Fig.2(c)) and the method based on asymmetric data (Fig.2(b)) is shown in Fig.7. According to this figure, the data distortion is not negligible in images' boundary area, especially when using asymmetric data. We subtracted the conventional method's data from the original picture to produce Fig. 7(c)) and subtracted our method's data from the orignal picture to produce Fig. 7(d). By comparing figures 7(c) and 7(d), we found our method was able to reduce the data distoration. The area of data distortion is related to a length of impulse responses of filters[5]. Namely, the area will be more expanded by using higher order filters. We have confirmed that our method is also effective for these filters, by using perfect reconstruction filters [11].

6. CONCLUSION

In this report, we have proposed a new filtering scheme for SBC which can reduce data distortion and avoid data increase with any type of linear phase FIR filter (see [10] for proof). Our method is based on considering the generating method of new periodic signals. The relationship between the input data and the output data though circular convolution is summarized in Table 1. Tables 3 and 4 show the results of down-sampling and up-sampling. The simulation results have shown the benefit of our method.

REFERENCES

[1] J.W.Woods and S.D.O'Neil, "Subband Coding of Images," IEEE Trans. Acoust., Speech and Signal Processing, ASSP-34, pp.1278-1288, (Oct. 1986)

[2] M.Vetterli, "Multi-dimensional Subband Coding; Some Theory and Algorithms," Signal Processing, 6, pp.97-112, (Apr. 1984)

[3] Didier Le Gall and Tabatabai "Subband Coding of Digital Images using Symmetric Short Kernel filters and Arithmetic Coding Techniques," Proc. ICASSP 88, pp.761-764, (1988)

[4] Hamid Gharavi and Ali Tabatabai "Subband Coding of Monochrome and Color Images," IEEE Trans. on Circuits and Systems, vol.35, No.2, (Feb.1988)

[5] Mark J.T.Smith and Steven L.Eddins "Analysis/ Synthesis Techniques for Subband Image Coding," IEEE Trans. Acoust., Speech, Signal Processing. vol.38, No.8, (Aug. 1990)

[6] Choon-Sung Kim, John Bruder, Mark J.T.Smith and Russell M.Mersereau "Subband Coding of Color Images using Finite State Vector Quantization," ICASSP 88, (Apr. 1988)

[7] Mark J.T. Smith and Steven L.Eddins "Subband Coding of Images with Octave Band Tree Structures," ICASSP 87, (Apr. 1987)

[8] D.Esteban and C.Galand "Application of Quadrature Mirror Filters to Splitband Voice Coding Schemes," Proc. of the IEEE International Conf. ASSP, Hartford, Connecticut, pp.191-195, (May 1977)

[9] L.R.Rabiner, B.Gold, "Theory and Application of Digital Signal Processing," Prentice-Hall, Inc. Englewood Cliffs (1975)

[10] K.Nishikawa,H.Kiya,M.Sagawa,"Property of Circular Convolution for Subband Image Coding",ICASSP (March 1992, to be published)

[11] H.Kiya,M.Yae,M.Iwahashi,"A Linear Phase Two Channel Filter Bank allowing Perfect Reconstruction", ISCAS, (May 1992, to be published)

Table 1. Types of filtered data.
Note: "*" indicates Smith's method.

input \ filter	1	2	3	4
A	A	C*	D	F*
B	B	B	E	E
C	C	A	F	D
D	D	F	A	C
E	E	E	B	B
F	F	D	C	A

Table 2. The only possible combinations as a decimation procedure.

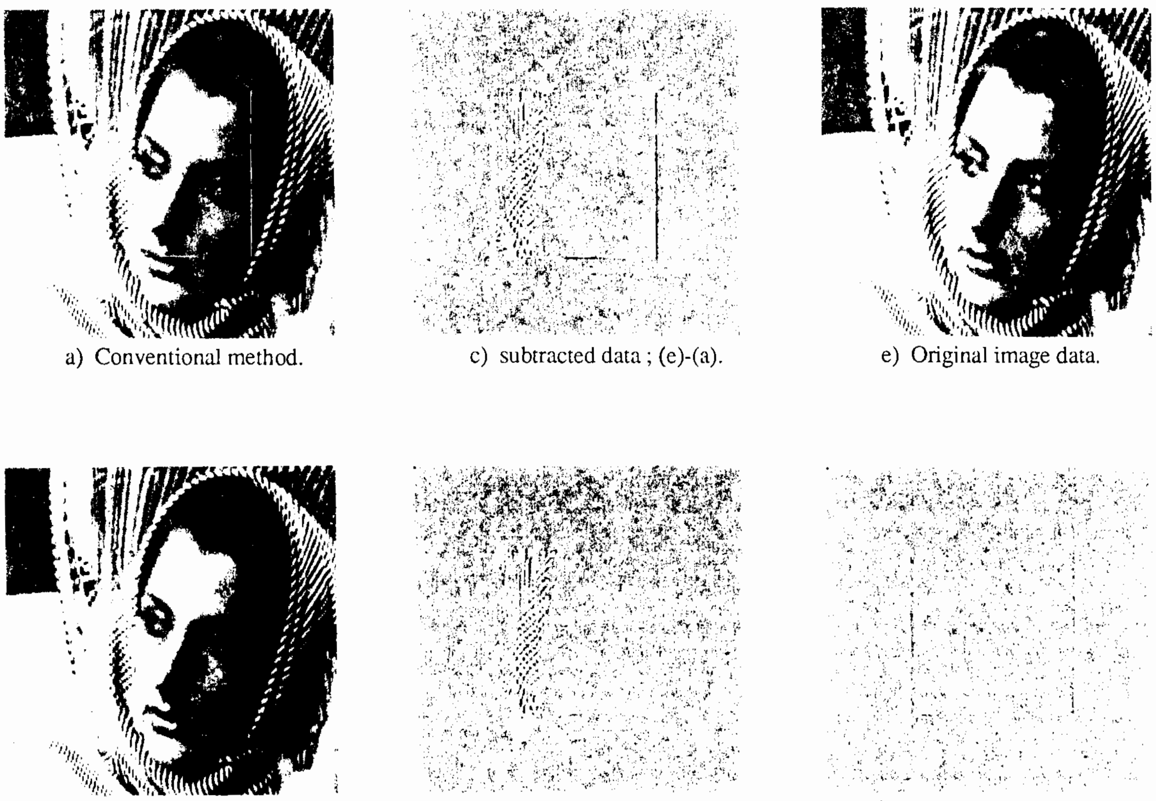
input \ filter	1	2	3	4
A	-	C*	-	F*
B	-	-	-	-
C	C	-	F	-
D	-	F	-	C
E	-	-	-	-
F	F	-	C	-

Table 3. Types of down-sampled data $\underline{z}(m)$.

$\underline{y}(m)$	M/2 ∈ even		M/2 ∈ odd	
	type-I	type-II	type-I	type-II
C	C	A	B	B
F	F	D	E	E

Table 4. Types of up-sampled data.

input; $\underline{z}'(m)$	A	B	C	D	E	F
output; $\underline{y}'(m)$	C	C	C	F	F	F



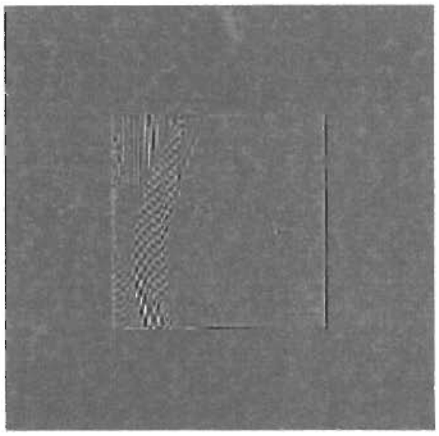
a) Conventional method.
c) subtracted data ; (e)-(a).
e) Original image data.
b) Proposed method.
d) subtracted data ; (e)-(b).
f) subtracted data ; (a)-(b).

Fig.7 Reconstructed images after quantization.

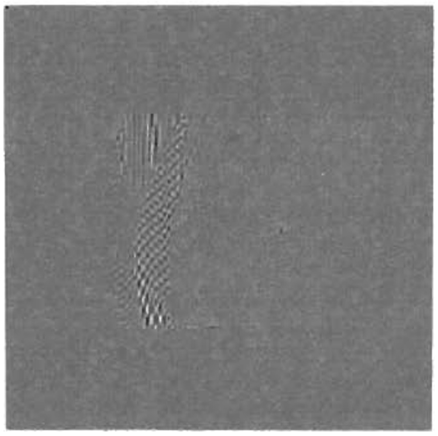
Image data are constructed of 200 x 200 pixels. Only 100 x 100 pixels at the center of the image data are filtered.

a) Conventional method.

b) Proposed method.

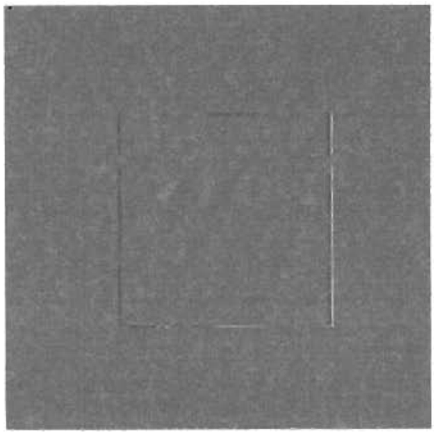


c) subtracted data ; (e)-(a).



d) subtracted data ; (e)-(b).

e) Original image data.



f) subtracted data ; (a)-(b).

Fig.7 Reconstructed images after quantization.

Image data are constructed of 200 x 200 pixels. Only 100 x 100 pixels at the center of the image data are filtered.