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A LINEAR PHASE TWO-CHANNEL FILTER BANK ALLOWING PERFECT RECONSTRUCTION

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Abstract

We propose a design method for a two-channel filter bank using linear phase filters. Our method yields a filter bank having good frequency response, though it is based on linear equations. It can lead a filter bank without multipliers, if we want such a bank.

1 INTRODUCTION

The purpose of this paper is to propose a design method for a two-channel filter bank using linear phase filters. This type of filter bank is especially important in splitting image signals into frequency bands for subband image cording. Because in such an application, it is necessary to use the combination of linear phase filters and symmetric image signal, namely linear phase signal to avoid the increase in image size caused by filtering, as noted in [1]-[4].

Work on filter banks was initiated by the introduction of the QMF(Quadrature Mirror Filter) concept[5][6]. This two-channel QMF bank which may be composed of linear phase filters can cancel the aliasing perfectly, while not solving the problem perfectly. The first perfect FIR solution was proposed by Smith and Barnwell[7]. However their solution does not include linear phase filters, that is, it must be composed of the minimum and maximum phase filters.

A perfect linear FIR solution for two-channel banks is respectively outlined in [8],[9] and [12]. However, the length of the linear-phase filter shown in [8] is confined to 2, 3, 4 or 5. Also even if the method suggested in [9] is used, it is difficult to design filters having good response, such as the chebyshev approximation. The reason is that the procedure is executed without considering frequency responses. Compared to the above methods[8][9], the method suggested in [12] yields better filters in the practical sense. However when using this method, the computational complexity for designing filter banks is increased, due to the fact that this procedure is based on non-linear optimization. This can not lead filters without multipliers, which can be implemented by a limited number of shifts and adds, and thus are computationally efficient, such as the SSKF (Symmetric Short Kernel Filter) bank[8]. For subband image coding, this type of filter is sometimes important because it reduces the hardware complexity.

Therefore in this paper, a new method for designing two-channel QMF banks is proposed. Our proposed procedure yields good filters, although it is based on linear equations. As a result, the computational complexity is almost same as that of ordinary filter designs. This method may also lead filters without multipliers, which enable us to use simple arithmetic operations.

2 DESIGN PROCEDURE

Fig.1 shows a block diagram of a two-channel filter bank. Then we obtain the input-to-output relationship of this system in the form[9]

$$Y(z) = [H_0(z)F_0(z) + H_1(z)F_1(z)]X(z) + [H_0(-z)F_0(z) + H_1(-z)F_1(z)]X(-z) \quad (1)$$

Suppose that (1) meets the following conditions, referred to as the necessary and sufficient condition for two-channel perfect QMF banks, namely perfect reconstruction

$$H_0(z)H_1(-z) - H_1(z)H_0(-z) = z^{-L} \quad (2)$$

$$F_0(z) = H_1(-z) \quad (3)$$

$$F_1(z) = -H_0(-z) \quad (4)$$

where L is an integer. Then (1) is reduced to (5).

$$Y(z) = z^{-L}X(z) \quad (5)$$

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Therefore, our purpose is to show how to design $H_0(z)$ and $H_1(z)$, which have linear phases and satisfy (2). The proposed method is summarized as follows.

Step 1 : We design a half-band lowpass filter $H_0(z)$(see Fig.2).

1-1 : Design a linear phase lowpass filter $H_0'(z)$ satisfying the following conditions:

* order D : Odd number
* passband edge ω_p : Arbitrary
* stopband edge ω_s : π

1-2 : Modify $H_0'(z)$ in the form

$$H_0(z) = 1/2[H_0'(z^2) + z^{-D}] \quad (6)$$

Step 2 : We choose the initial solution $H_1'(z)$ for designing a high pass filter as

$$H_1'(z) = 1 \quad (7)$$

Substituting (6) and (7) into (2), it is easily shown that (7) meets the condition for perfect reconstruction. That is, we obtain

$$H_0(z)H_1'(-z) - H_1'(z)H_0(-z) = z^{-D} \quad (8)$$

Step 3 : We determine a high pass filter $H_1(z)$ in the form

$$H_1(z) = H_1'(z)z^{-J} - f(z)H_0(z) \quad (9)$$

where $f(z)$ and J satisfy the three conditions shown as ;

1. $H_1(z)$ is a perfect solution if

$$J : \textit{Even number} \quad (10)$$

$$f(z) = f(-z) : \textit{Arbitrary function} \quad (11)$$

2. Under(10)(11), $H_1(z)$ has a linear phase if

$$J = (2D + 2K)/2 \quad (12)$$

$$K : \textit{Arbitrary odd number} \quad (13)$$

where $2K$ is the order of $f(z)$ having a linear phase.

3. Under(10)-(13), $H_1(z)$ is a high pass filter if

$$f(z)H_0(z): \textit{Low pass filter having unit gain} \quad (14)$$

Now,we will explane why the above three conditions are needed , respectively. Condition 1. may be explained as follows, as noted in [10]. By replacing $H_1(z)$ by $H_1'(z)$ in (2), we obtain

$$H_0(z)H_1'(-z) - H_0(-z)H_1'(z) = z^{-L} \quad (15)$$

Next multiplying (15) by z^{-J},

$$H_0(z)H_1'(-z)(-z)^{-J} - H_0(-z)H_1'(z)z^{-J} = z^{-(J+L)} \quad (16)$$

where J is an even number. By introduction of a function $f(z) = f(-z)$, the above equation is rewritten as

$$H_0(z)[H_1'(-z)(-z)^{-J} - f(-z)H_0(-z)] - H_0(-z)[H_1'(z)z^{-J} - f(z)H_0(z)] = z^{-(J+L)} \quad (17)$$

or equivalently

$$H_0(z)H_1(-z) - H_0(-z)H_1(z) = z^{-(J+L)} \quad (18)$$

where

$$H_1(z) = H_1'(z)z^{-J} - f(z)H_0(z) \quad (9)$$

Therefore, $H_1(z)$ in (9) will be a perfect solution if $H_1'(z)$ is a perfect solution.

Note that the $f(z)H_0(z)$ has $2D + 2K + 1$ impluse response under condition 2, and the center of symmetry of them occureds at the $J = (2D + 2K)/2$th sequense. Thus the $H_1(z) = z^{-J} - f(z)H_0(z)$ shown in (9) has symmetric impulse responses. (9) also lead to a hipass filter if the $f(z)H_0(z)$ is a lowpass filter, because of $H_1(z) + f(z)H_0(z) = z^{-J}$.

3 DESIGN EXAMPLES

3.1 Example 1

We will consider a simple example. We choose the lowpass filter $H_0'(z)$ with order $D = 1$ as

$$H_0'(z) = 1/2(1 + z^{-1}) \quad (19)$$

From (6), we get

$$H_0(z) = 1/4(1 + 2z^{-1} + z^{-2}) \quad (20)$$

Using $f(z) = f(-z) = 1/2(1 + z^{-2}), J = K + D = 2$ and $H_1'(z) = 1$, from (9) we get

$$H_1(z) = -1/8(1 + 2z^{-1} - 6z^{-2} + 2z^{-3} + z^{-4}) \quad (21)$$

Fig.3 shows the frequency responses of $H_0(z)$ and $H_1(z)$, respectively. It should be noted that we can also obtain other solutions, for example by choosing $H_0(-z)$ for a highpass filter and $H_1(-z)$ for a lowpass filter, or by using other $f(z)$ and J. When choosing

$H_0(-z)$ and $H_1(-z)$ in this example, we can lead the filter bank referred to as the SSKF (Symmetric Short Kernel Filter) bank[8]. Similarly, it should be noted that this procedure easily yields filter banks without multipliers, if $H_0(z)$ is designed as multiplierless and $f(z)$ without multipliers is used.

3.2 Example 2

Next, let us consider a filter bank with higher order. Similarly, first we design a lowpass filter $H_0(z)$. Fig.4 illustrates the frequency response of $H_0(z)$ designed by using Remez algorithm[11]. Next in order to design a highpass filter $H_1(z)$, we must determine $f(z)$. For example we may choose $f(z)$ from Table 1. The table gives linear phase maximally flat FIR functions with odd order, which have frequency responses as shown in Fig.5, respectively[13]. If we choose $f(z)$ with order $2K = 30$, then we get the high pass filter $H_1(z)$ shown in Fig.4. It should be noted that we may choose other $f(z)$, such as the one designed by using Remez algorithm[11], or some window functions.

4 CONCLUDING REMARKS

We have described a perfect-reconstruction two-channel QMF bank, in which the analysis and synthesis filters have linear phases. First, a technique for designing filter banks has been presented. Next the results of two examples have demonstrated that the method can lead good filters although its procedure is very simple, and also this yields filters without multipliers such as the SSKF bank.

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Table 1 Linear-phase maximally flat functions with odd oder. $f(z^{1/2})$

n	K=3	K=7	K=15
0	-0.06250014691797	$-2.4414527110208\times10^{-3}$	$-6.4095624961802\times10^{-6}$
1	0.56249992572967	2.3925717063599 10^{-2}	$1.1061853815747\times10^{-4}$
2		-0.1196290534547	$-9.1532677863918\times10^{-4}$
3		0.5981444440123	$4.8476788861765\times10^{-3}$
4			$-1.8698403022615\times10^{-2}$
5			$5.7590858009221\times10^{-2}$
6			-0.1599749110378
7			0.6170454438427

note: $h(n)=h(K-n)$, K:order of $f(z^{1/2})$

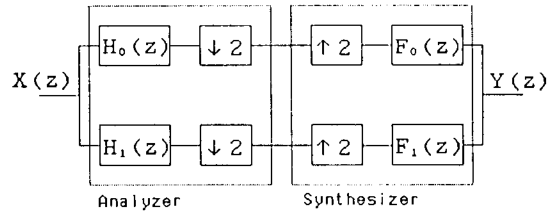
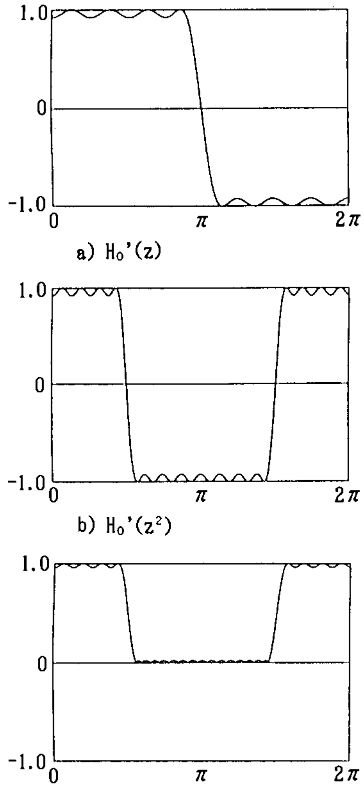


Fig.1 Two-channel filter bank



a) $H_0'(z)$

b) $H_0'(z^2)$

c) $1/2[H_0'(z^2)+z^{-D}]$

Fig.2 A design procedure for linear-phase half band filters.

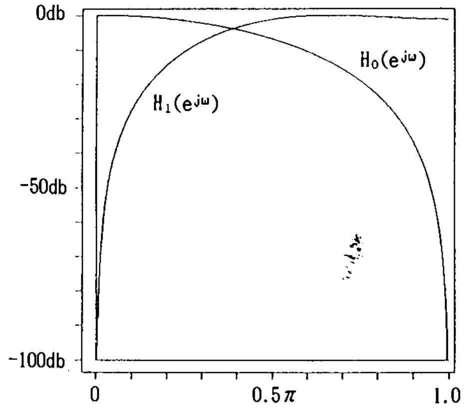


Fig.3 Frequency responses (example 1)
$H_0(z)$ has 3 taps. $H_1(z)$ has 5 taps.

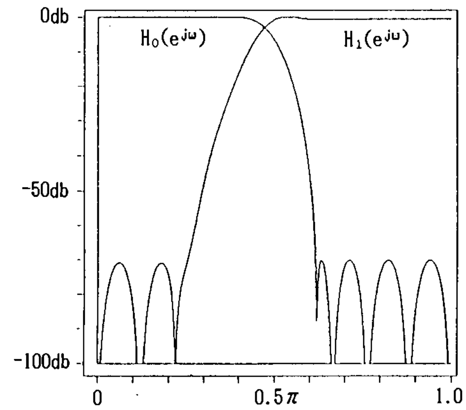


Fig.4 Frequency responses (example 2)
$H_0(z)$ has 31 taps. $H_1(z)$ has 61 taps.

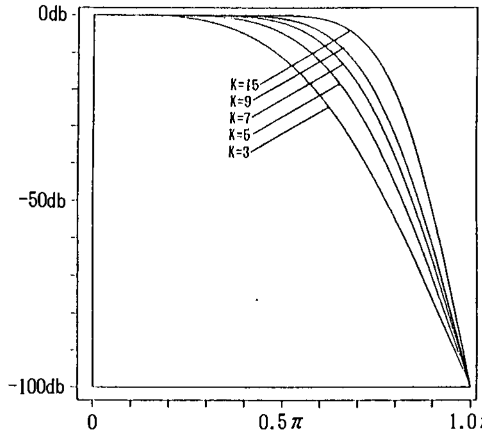


Fig.5 Frequency responses of the maximally flat function. $f(z^{1/2})$