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LOSSLESS MULTI CHANNEL PREDICTIVE CODING FOR IMAGES

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ABSTRACT

This report proposes a new lossless coding of images. The decoder can expand a rough image from a part of the bit stream (compressed data) and also it can expand the original image from the rest. This functionality is useful for progressive transmission of images or browsing images in a huge database. The new method is based on a reversible wavelet (RWT) and a lossless multi-channel prediction (LMP). Filter coefficients of the LMP are optimized for each input image so that the total bit rate is minimized. Experimental results indicate its effectiveness for use of lossless image coding. Entropy rate is reduced by 0.35 bpp in average for some images.

1. INTRODUCTION

For lossless coding of still images, lossless JPEG (L-JPEG) [1] based on DPCM and arithmetic coding is widely used at present. Due to fixed value of coefficients, it can not be adaptive to images of various characteristics. JPEG-LS [2] as the next international standard features context modeling and a non linear predictor referred to MED (Median Edge Detector) in LOCO [3]. Some other non linear predictors such as GAP (Gradient Adaptive Prediction) in CALIC [5] and FIR-Boolian [4] are developed to afford locally varying statistics (edges) and confirmed to be superior to the conventional L-JPEG. However these new predictors are not suitable for progressive transmission because they contain no *anti-aliasing filter* and the *aliasing* is caused in low-resolution images.

On the other hand, reversible wavelet transform (RWT) inherently has advantage for the progressive coding as it produces low-resolution images as a part of its encoding procedure. So far various RWT have been proposed such as SP transform [6] and TS transform in CREW [7]. Some generalized RWT based on the lifting structure [8] have also been proposed lately [9]. However there still exists remaining correlation in band signals of the RWT and it has been left not being utilized.

In this report, we investigate still remaining correlation in sub-bands of the RWT first. Next, we utilize this with the lossless multi-channel prediction (LMP) based on a non separable two dimensional (2D) filter bank [10][11]. Furthermore, we optimize parameters of the LMP to minimize the total bit rate. We also confirm its effectiveness for images with some experiments.

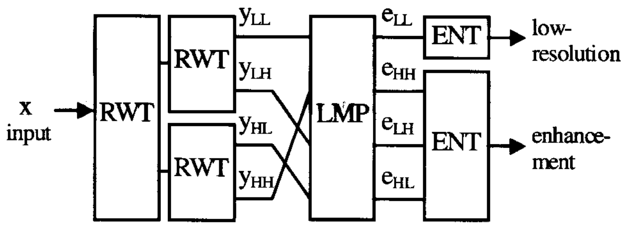

Figure 1. Proposed encoder. RWT, LMP and ENT denote the reversible Wavelet, the lossless multi-channel prediction and the entropy coding respectively.

2. REVERSIBLE WAVELETS

2.1 The problem addressed here

Among various kinds of RWT, S transform attracts many researchers' attention due to short tap filters (only 2 tap), orthogonal characteristics, no increase in bit depth of its low frequency band signal, etc. For example, TS transform in CREW [7] can be expressed in the form of including the S transform in it. The TS transform employs a prediction from low band to high band (*"inter"* band prediction). This is essentially equivalent to using a long tap filter in high. However it excludes *"intra"* band prediction (a prediction from the band of itself). SP transform [6] adopts the inter band prediction and also the intra band prediction. However filter coefficients are fixed and it cannot escape from restrictions of *"separable"* 2D filtering [10][11].

To solve this problem, we utilize remaining correlation existing in sub-bands of the S transform (a kind of RWT) applying *"inter / intra"* band predictions with the LMP (a *"non separable"* 2D filter). Figure 1 illustrates the proposed encoding procedure. We also optimize parameters (filter coefficients) of the LMP for lossless data compaction in **3.**.

2.2 Remaining correlation of the S transform

The S transform decomposes an M point input signal $x(m)$, $m=0,1,\cdots,M-1$ into two band signals $y_L(n)$ and $y_H(n)$, $n=0,1,\cdots,N-1$, $(N=M/2)$ by

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The inverse procedure of equation (3) defined by

$$y_p(n_1,n_2)=e_p(n_1,n_2)-R\left[\sum_{q=0}^{p} H_{qp}[y_q(n_1,n_2)]\right] \quad (5)$$
$$p=0,1,2,3$$

is applied in the decoder. It is assumed that bit error in transmission channel is 100% corrected in a proper way.

3.2 Optimization of the coefficients

We need to determine filter coefficients so that the total bit rate of the prediction errors becomes minimum. In general, the bit rate is in proportion to logarithm of the error variance [12], namely,

$$B_p = \log_2 \gamma\sqrt{\| e_p \|^2} \quad , \quad p \in \{0,1,2,3\} \quad (6)$$

where

$$\| e_p \|^2 = \sum_{n_2=0}^{N_2-1}\sum_{n_1=0}^{N_1-1} e_p^2(n_1,n_2) \quad (7)$$

Therefore our purpose is to minimize the total bit rate

$$B_{SBC} = \frac{1}{4}\sum_{p=0}^{3} B_p = \log_2 \gamma \sqrt{\left\{\prod_{p=0}^{3} \| e_p \|^2\right\}^{1/4}} \quad [bit] \quad (8)$$

In this report, we minimize the variance of e_{HL} at first, and then, e_{LH}, e_{HH} and e_{LL}. This allows us to use a simple calculation procedure well known as the least square auto regression algorithm (LS-AR). Ignoring tiny effect of R[ ] and substituting equation (3) and (4), equation (7) becomes

$$\| e_p \|^2 = \sum_{n_2=0}^{N_2-1}\sum_{n_1=0}^{N_1-1} \quad (9)$$
$$\left\{ y_p(n_1,n_2) + \sum_{q=0}^{3}\sum_i\sum_j c_p^{(q,i,j)} y_q(n_1+i,n_2+j) \right\}^2$$

As a result of applying LS-AR to equation (9), the optimized coefficients are calculated as a solution of the linear equations

$$\sum_{q=0}^{3}\sum_i\sum_j c_p^{(q,i,j)} \cdot A_p^{(r,k,l\leftrightarrow q,i,j)} = -B_p^{(r,k,l)} \quad (10)$$
$$(p,r) \in \{0,1,2,3\} \quad , \quad (k,l) \in \{0,\pm 1,\pm 2,\cdots\}$$

where

$$A_p^{(r,k,l\leftrightarrow q,i,j)} = \sum_{n_2=0}^{N_2-1}\sum_{n_1=0}^{N_1-1} y_r(n_1+k,n_2+l)\, y_q(n_1+i,n_2+j)$$

$$B_p^{(r,k,l)} = \sum_{n_2=0}^{N_2-1}\sum_{n_1=0}^{N_1-1} y_r(n_1+k,n_2+l)\, y_p(n_1,n_2)$$

The optimized coefficients should be shared between the encoder and the decoder. Therefore we need to include the coefficients into the bit stream as over head information. However the proposed encoder can afford various characteristics of images since the parameters are optimized to an input image.

4. SIMULATION RESULTS

Table 2 indicates effectiveness of combination of the RWT and the LMP *–the proposed method–* in respect of first order entropy rate defined by

$$H = -\sum_s P_s \log_2 P_s \quad (11)$$

where P_s indicates probability of a symbol "s". Some of existing RWT, such as TS transform in CREW [7] and SP transform [6], are also attached as reference. We used luminance of each images (8 bpp) being removed its average. The original image is decomposed into four bands (one stage decomposition). As the lowest band (LL) of S transform and TS transform is not compressed, we applied DPCM "y(n)=x(n)-x(n-1)" to it in horizontal and in vertical direction.

An example of the optimized filter coefficients for "Lena" are listed in table 3 where predicted errors are calculated by

$$\begin{aligned} E_{HL} = Y_{HL} + R[\; & c_1 HL[-1,0] + c_2 HL[0,-1] \\ & + c_3 LL[-1,-1] + c_4 LL[1,-1] + c_5 LL[-1,0] \\ & + c_6 LL[1,0] + c_7 LL[-1,1] + c_8 LL[1,1] \\ & + c_9 HH[0,-1] + c_{10} HH[0,1] \;] \end{aligned} \quad (12)$$

$$\begin{aligned} E_{LH} = Y_{LH} + R[\; & c_1 LH[-1,0] + c_2 LH[0,-1] \\ & + c_3 LL[-1,-1] + c_4 LL[-1,1] + c_5 LL[0,-1] \\ & + c_6 LL[0,1] + c_7 LL[1,-1] + c_8 LL[1,1] \\ & + c_9 HH[-1,0] + c_{10} HH[1,0] \;] \end{aligned} \quad (13)$$

$$\begin{aligned} E_{HH} = Y_{HH} + R[\; & c_5 HH[-1,0] + c_6 HH[0,-1] \\ & + c_1 LL[-1,-1] + c_2 LL[1,-1] \\ & + c_3 LL[-1,1] + c_4 LL[1,1] \;] \end{aligned} \quad (14)$$

$$\begin{aligned} E_{LL} = Y_{LL} + R[\; & c_1 LL[-1,0] + c_2 LL[0,-1] \\ & + c_3 LL[-1,-1] + c_4 LL[1,-1] \;] \end{aligned} \quad (15)$$

and

$$E_p = e_p(n_1,n_2), \qquad Y_p = y_p(n_1,n_2),$$
$$P[i,j] = y_p(n_1+i,n_2+j), \quad p \in \{LL,HL,LH,HH\}$$

Effectiveness of our method was confirmed by 0.35 bpp in average comparing to "S Transform + DPCM". For "Lena", it was 0.36 bpp. Prior to decoding the compressed data, we need to transmit all of thirty coefficients in table 3 as side information. In this experiment, coefficient values are truncated into 8 bit. It costs 0.001 bpp in case of 512 x 480 pixel image.

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$$\begin{pmatrix} y_L(n) \\ y_H(n) \end{pmatrix} = \begin{pmatrix} F\left[\dfrac{x(2n)+x(2n+1)}{2}\right] \\ x(2n)-x(2n+1) \end{pmatrix} \qquad (1)$$

F[ ] denotes *flooring* into an integer value and x(m) also takes an integer value. Applying equation (1) to the original image $x(n_1,n_2)$ in two directions, we get four sub-bands denoted by $y_{LL}(n_1,n_2)$, $y_{HL}(n_1,n_2)$, $y_{LH}(n_1,n_2)$ and $y_{HH}(n_1,n_2)$. For example, $y_{HL}(n_1,n_2)$ indicates horizontally high passed (H) and vertically low passed (L).

Calculating the inter / intra band correlation coefficients

$$C_q^{(p,i,j)} = \left| \frac{\sum_{n_2=0}^{N_2-1}\sum_{n_1=0}^{N_1-1} y_q(n_1,n_2)\, y_p(n_1+i,n_2+j)}{\sum_{n_2=0}^{N_2-1}\sum_{n_1=0}^{N_1-1} y_q^2(n_1,n_2)} \right| \times 100 \quad [\%] \qquad (2)$$

$$(p,q) \in \{LL, HL, LH, HH\}$$

of some images, we investigated the remaining correlation of the sub-bands. Table 1 is an example for luminance of "Lena". There exists remaining correlation commonly high in images, such as

$C_{HL}^{(HL,0,\pm1)}$, $C_{LH}^{(LH,\pm1,0)}$

$C_{HL}^{(LL,\pm1,0)}$, $C_{LH}^{(LL,0,\pm1)}$, $C_{HH}^{(HL,0,\pm1)}$, $C_{HH}^{(LH,\pm1,0)}$

For example, $C_{HL}^{(LL,\pm1,0)}$ indicates correlation of "HL" to adjacent pixels of "LL" in right hand and left hand. The first row means *"intra band"* correlation and the second is *"inter band"*. We utilize them in **3.** for higher compaction efficiency.

The lowest resolution signal $y_{LL}(n_1,n_2)$ is a down-scaled version of an input image. Comparing figure 2(a) and (b), we can confirm that unnecessary *"aliasing"* is significantly reduced by the *anti-aliasing filter* in the S transform. This signal is transmitted and decoded at first and used as a rough image (thumb-nail) in the progressive transmission.

3. LOSSLESS MULTI-CHANNEL PREDICTION

3.1 Signal processing procedure

In this report, we adopt the losssless multi-channel prediction (LMP) [10][11] to make the best use of the inter / intra band correlation confirmed in **2.2.** The LMP is a multi channel non separable two dimensional filter bank as illustrated in figure 3 and it affects on an input signal as a linear predictor. Here in after, we denote {HL, LH, HH, LL} as {3, 2, 1, 0} respectively. Four types of input signals $y_p(n_1,n_2)$ are predicted each other by

$$e_p(n_1,n_2) = y_p(n_1,n_2) + R\left[\sum_{q=0}^{p} H_{qp}[y_q(n_1,n_2)]\right] \qquad (3)$$

$$p = 3,2,1,0$$

R[ ] denotes *"rounding"* into an integer and H_{qp}[ ] is an FIR filter described with $c_p^{(q,i,j)}$ as

$$H_{qp}[y_q(n_1,n_2)] = \sum_j \sum_i c_p^{(q,i,j)}\, y_q(n_1+i, n_2+j) \qquad (4)$$

Then we get output signals $e_p(n_1,n_2)$, p=3,2,1,0, as prediction errors with reduced variances.

(a)

(b)

Figure 2. Low resolution images of "Barbara". (a): Sub-sampled without any filter. (b): Anti aliasing filter of the S transform is applied before sub-sampling.

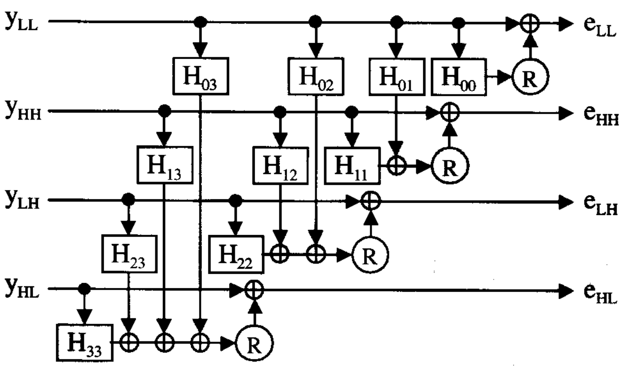

Figure 3. Lossless multi channel prediction (LMP).

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5. SUMMARY

We have proposed a new lossless progressive coding of digital image data. The method is based on RWT (reversible wavelet transform) and LMP (lossless multi channel prediction). The former is for reduction of aliasing in low-resolution images and latter is for utilization of remaining correlation of the RWT. We have optimized filter coefficients of the LMP for lossless coding by minimizing prediction error variances. Effectiveness of the method has also been confirmed as 0.35 bpp of entropy rate reduction. We are now expanding it into a locally adaptive one.

6. REFERENCES

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Table 1 Remaining correlation $C_q^{(p,i,j)}$ of "Lena".
(8 bit Luminance, 512x480 pixels)

	$C_{HL}^{(LL,i,j)}$			$C_{HL}^{(HL,i,j)}$			$C_{HL}^{(LH,i,j)}$			$C_{HL}^{(HH,i,j)}$		
j \ i	-1	0	1	-1	0	1	-1	0	1	-1	0	1
-1	77	14	62	4	59	15	11	3	20	5	24	7
0	90	0	88	2	100	2	2	11	2	5	0	4
1	64	13	74	15	60	4	19	2	11	7	24	6
	$C_{LH}^{(LL,i,j)}$			$C_{LH}^{(HL,i,j)}$			$C_{LH}^{(LH,i,j)}$			$C_{LH}^{(HH,i,j)}$		
-1	60	95	32	24	5	40	7	7	29	0	9	4
0	26	4	34	5	23	5	31	100	31	33	0	32
1	40	103	69	41	5	23	30	7	7	3	9	1
	$C_{HH}^{(LL,i,j)}$			$C_{HH}^{(HL,i,j)}$			$C_{HH}^{(LH,i,j)}$			$C_{HH}^{(HH,i,j)}$		
-1	20	6	35	10	41	12	1	7	2	5	20	16
0	5	20	4	8	0	9	27	0	27	24	100	24
1	35	5	21	11	41	9	3	7	0	16	20	5
	$C_{LL}^{(LL,i,j)}$			$C_{LL}^{(HL,i,j)}$			$C_{LL}^{(LH,i,j)}$			$C_{LL}^{(HH,i,j)}$		
-1	92	97	94	4	1	3	2	3	1	1	0	1
0	94	100	94	4	0	4	1	0	1	0	1	0
1	94	97	92	3	1	4	1	2	1	1	0	1

Table 2 First order entropy rate of the methods.
(8 bit Luminance, Single stage).

	S Trans. + DPCM	SP Trans.	TS Trans. + DPCM	**S Trans. + LMP**
Aerial	6.25	5.97	5.97	**5.79**
Barbara	5.47	5.24	5.12	**5.03**
Church	6.56	6.38	6.38	**6.20**
Couple	4.60	4.43	4.40	**4.38**
Girl	4.98	4.89	4.72	**4.68**
Lena	4.73	4.59	4.46	**4.37**
Moon	5.26	5.40	5.14	**4.98**
Average (difference)	5.41 (0.0)	5.27 (-.14)	5.17 (-.24)	**5.06 (-.35)**

Table 3 Optimized coefficients of LMP for "Lena" (8 bit Luminance, 512x480 pixels).

Band	c1	c2	c3	c4	c5	c6	c7	c8	c9	c10
HL	0.2031	-0.3281	0.0625	-0.0781	-0.2500	0.2813	-0.0469	0.0156	0.2188	-0.2031
LH	-0.0469	0.2344	0.0000	-0.0156	-0.3281	0.3438	0.0156	-0.0156	0.0781	-0.0781
HH	-0.1250	0.1406	0.1406	-0.1406	0.2188	0.2031				
LL	-0.5000	-0.5313	0.2656	-0.2344						

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