

## ADAPTIVE MULTI-CHANNEL PREDICTION FOR LOSSLESS SCALABLE CODING

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### ABSTRACT

In this report, a new lossless coding of images is proposed. The method has the "scalability", namely, it can expand a rough image from a part of the bit stream (compressed data) and also expand the original image from the rest. The new method is adaptive to locally varying characteristics such as "edge" in the input image. A new lossless "anti-aliasing" filter is designed to reduce the "aliasing" in the rough image. A lossless multi channel non separable 2D filter bank is applied and its filter coefficients are adaptively optimized for each of the pixels according to local characteristics of the image. It is not necessary to include the coefficients into the bit stream since both of the encoder and the decoder employ the same procedure to calculate the coefficients. Experimental results are indicated to confirm its effectiveness.

### 1. INTRODUCTION

Up to now, various types of lossless coding have been proposed. "extrapolation" in single channel such as differential pulse code modulation (DPCM) has been widely used for lossless coding [1]. Recently, "interpolation" has also been applied to the lossless coding in the form of multi channel [2-7]. This form is essentially same as a multi-channel filter bank with linear filters. Non-linear filters are also applied to utilize local characteristics of images such as "edges" [8-10].

In this report, we aim at creating a new "adaptive lossless scalable" coding. SP transform [6] can be used as a lossless and scalable coding. It can expand a rough image from a part of the compressed data and also expand the image exactly same as the original image (without any loss) from the rest. However it has a structural restriction due to cascading one dimensional filters [7]. As a result, diagonal correlation of the image is not utilized.

Lossless multi-channel prediction (LMP), which is based on a non separable two dimensional multi-channel filter bank, is proposed to avoid this problem [7]. Its parameters are optimized to meet global characteristics of an input image. However, optimized filter coefficients should be transmitted to the decoder as side information and it is not adaptive to local characteristics such as "edges".

This report proposes a new lossless scalable coding adaptive to the local characteristics of an image without any side information. We design a new lossless "anti-aliasing" filter for reduction of the "aliasing" in rough images in 2.1. The LMP [7] is detailed in

2.2. We introduce the "context modeling" [8] to remove the side information and to utilize local characteristics of an input image in 2.3. A procedure to calculate filter coefficients is summarized in 2.4. We confirmed effectiveness of our method in 3.

## 2. ADAPTIVE LOSSLESS SCALABLE CODING FOR IMAGES

### 2.1 Lossless anti-aliasing filter

To have a good quality of rough images (low resolution images) in progressive data transmission, it is desirable to reduce the "aliasing" caused from sub-sampling of the image. For this purpose, we design a new lossless anti-aliasing filter (LAAF).

Figure 1 illustrates a two point lossless transform [11]. Denoting a 1D input signal  $x(m)$ ,  $m=0,1,\dots,M-1$ , it decomposes  $x(m)$  into two band signals  $y(2n)$  and  $y(2n+1)$  by

$$\begin{pmatrix} y(2n) \\ y(2n+1) \end{pmatrix} = \begin{pmatrix} x(2n) + F[\beta x'(2n+1)] \\ x'(2n+1) + F[\gamma y(2n)] \end{pmatrix} \quad (1)$$

where

$$\begin{aligned} x'(2n+1) &= x(2n+1) + F[\alpha x(2n)] \\ n &\in [0, N), \quad N = M/2 \end{aligned}$$

$F[\ ]$  denotes a flooring operation (a truncation into integer value). Neglecting tiny effect of  $F[\ ]$ , equation (1) is expressed by

$$\begin{pmatrix} y(2n) \\ y(2n+1) \end{pmatrix} = \begin{pmatrix} 1 & 0 \\ \alpha & 1 \end{pmatrix} \begin{pmatrix} 1 & \beta \\ 0 & 1 \end{pmatrix} \begin{pmatrix} 1 & 0 \\ \gamma & 1 \end{pmatrix} \begin{pmatrix} x(2n) \\ x(2n+1) \end{pmatrix} \quad (2)$$

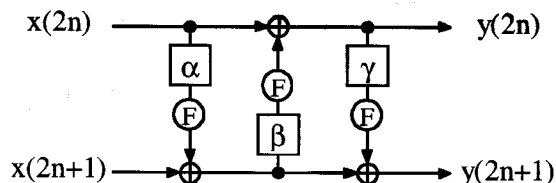
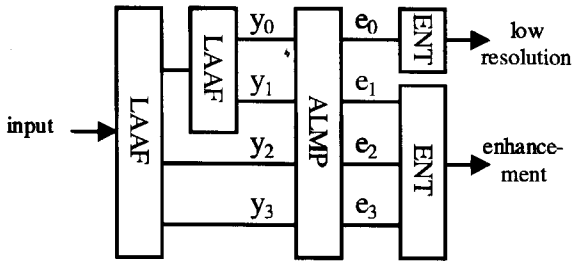


Figure 1. Two point lossless transform [14] for lossless anti-aliasing filter (LAAF).



**Figure 2.** Adaptive lossless scalable coding (encoder) based on lossless anti-aliasing filters (LAAF) and an adaptive lossless multi-channel prediction (ALMP). Prediction errors are encoded by entropy coders (ENT).

which is equivalent to

$$\begin{pmatrix} y(2n) \\ y(2n+1) \end{pmatrix} = \begin{pmatrix} a & b \\ c & d \end{pmatrix} \begin{pmatrix} x(2n) \\ x(2n+1) \end{pmatrix} \quad (3)$$

where

$$(\alpha \ \beta \ \gamma) = \begin{pmatrix} d-1 & b & a-1 \\ b & & b \end{pmatrix}, \quad ad - bc = 1$$

We want to make  $y(2n)$  an anti-aliasing filtered version of  $x(m)$  and  $y(2n+1)$  an all-pass filtered, namely,

$$\begin{pmatrix} L(x) \\ A(z) \end{pmatrix} = \begin{pmatrix} a & b \\ c & d \end{pmatrix} \begin{pmatrix} 1 \\ z \end{pmatrix} \rightarrow \begin{pmatrix} \text{low pass filter} \\ \text{all pass filter} \end{pmatrix} \quad (4)$$

To attain equation (4), we set

$$(a \ b) = (1/2 \ 1/2) \quad (5)$$

and

$$\begin{aligned} \text{minimize} \quad I &= \int_0^\pi |A(e^{j\omega}) - e^{-j\omega}|^2 d\omega \\ \text{subject to} \quad ad - bc &= 1 \end{aligned} \quad (6)$$

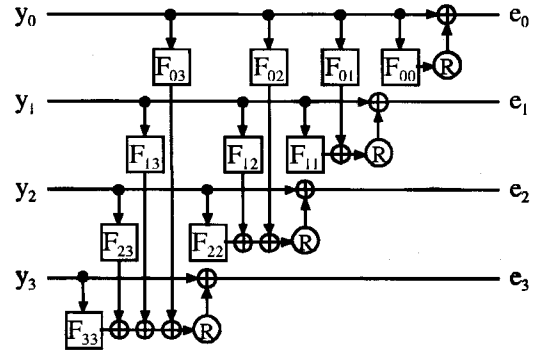
As a result, we get

$$(c \ d) = (-1/2 \ 3/2) \quad (7)$$

We apply equation (1) with (5) and (7) to a 2D input image  $x(m_1, m_2)$  in both horizontal and vertical to reduce the "aliasing" at first. Then, we use the adaptive LMP (ALMP) to utilize correlation between pixels.

Figure 2 illustrates the encoder of the proposed method. In this figure, output signals of the LAAF become

$$\begin{aligned} \begin{pmatrix} y_0(n_1, n_2) \\ y_1(n_1, n_2) \end{pmatrix} &= \begin{pmatrix} a & b \\ c & d \end{pmatrix} \begin{pmatrix} x'(2n_1, 2n_2) \\ x'(2n_1, 2n_2+1) \end{pmatrix} \\ \begin{pmatrix} y_2(n_1, n_2) \\ y_3(n_1, n_2) \end{pmatrix} &= \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix} \begin{pmatrix} x'(2n_1+1, 2n_2) \\ x'(2n_1+1, 2n_2+1) \end{pmatrix} \end{aligned} \quad (8)$$



**Figure 3.** Adaptive lossless multi channel prediction (ALMP). Coefficients of the FIR filters  $F_{ij}$  are adaptively calculated for each pixel.

where

$$\begin{pmatrix} x'(2n_1, m_2) \\ x'(2n_1+1, m_2) \end{pmatrix} = \begin{pmatrix} a & b \\ c & d \end{pmatrix} \begin{pmatrix} x(2n_1, m_2) \\ x(2n_1+1, m_2) \end{pmatrix}$$

and

$$\begin{pmatrix} y_0(n_1, n_2) & y_3(n_1, n_2) \\ y_2(n_1, n_2) & y_1(n_1, n_2) \end{pmatrix} = \begin{pmatrix} y(2n_1, 2n_2) & y(2n_1+1, 2n_2) \\ y(2n_1, 2n_2+1) & y(2n_1+1, 2n_2+1) \end{pmatrix}$$

In this equation,  $y_0(n_1, n_2)$  is the rough image, transmitted, decoded at first and used as a thumb nail in a progressive transmission application. This signal is down scaled with the anti aliasing filter  $L(z)$  in equation (4).

## 2.2 Lossless multi-channel prediction

After the LAAF, we apply lossless multi-channel prediction (LMP) [7] to minimize the total bit rate of the image utilizing correlation of the pixels. Figure 3 illustrates a block diagram of the LMP which is a non separable 2D lossless filter bank and produces prediction errors

$$e_p(n_1, n_2) = y_p(n_1, n_2) + R[\hat{y}_p] \quad , \quad p \in [0,3] \quad (9)$$

where  $R[\ ]$  denotes rounding to integer. In this report, we use predictions below.

$$\hat{y}_3 = a_3 \{Y[1,0] + Y[-1,0]\} + b_3 \{Y[0,1] + Y[0,-1]\} + c_3 \{Y[1,1] + Y[-1,-1]\} + d_3 \{Y[1,-1] + Y[-1,1]\} \quad (10)$$

$$\hat{y}_2 = a_2 \{Y[1,0] + Y[-1,0]\} + b_2 \{Y[0,1] + Y[0,-1]\} + c_2 \{Y[2,1] + Y[-2,-1] + Y[1,2] + Y[-1,-2]\} + d_2 \{Y[2,-1] + Y[-2,1] + Y[1,-2] + Y[-1,2]\} \quad (11)$$

$$\hat{y}_1 = a_1 Y[-2,0] + b_1 Y[0,-2] + c_1 \{Y[1,1] + Y[-1,-1]\} + d_1 \{Y[-1,1] + Y[1,-1]\} \quad (12)$$

$$\hat{y}_0 = a_0 Y[-2,0] + b_0 Y[0,-2] + c_0 Y[-2,-2] + d_0 Y[2,-2] \quad (13)$$

where

$$Y[i, j] = y(n_1 + i, n_2 + j)$$

The coefficients

$$\mathbf{V}_p = (a_p \ b_p \ c_p \ d_p) \quad (14)$$

are determined so that

$$J = \sum_{n_1, n_2 \in C_p} e_p^2(n_1, n_2) \rightarrow \text{minimum} \quad (15)$$

where  $C_p = \{\text{whole image area}\}$ . Applying the LMS (least mean square) algorithm to equation (15), the optimized coefficients are calculated as a solution of the Yule Walker's equation [14]. The coefficients are included into compressed data as side information in the existing LMP [7].

### 2.3 Context modeling

To remove the side information, we introduce "context modeling" [10] into the LMP and renew coefficients pixel to pixel using neighboring pixels. As a result, this adaptive LMP (ALMP) can afford locally changing characteristics such as "edges" without any over head information.

First, we categorize each of the pixels to be encoded into some groups according to status of its neighboring pixels (context). Letting an integer "k" the "context number", "k" is determined by a set of neighboring pixels (referred to "musk set"). For example, we use the musk set

$$\Theta_3 = \{Y[1,0], Y[-1,0], Y[0,1], Y[0,-1], Y[1,1], Y[-1,-1], Y[1,-1], Y[-1,1]\} \quad (16)$$

$$\Theta_2 = \left\{ \begin{array}{l} Y[1,0], Y[-1,0], Y[0,1], Y[0,-1], \\ (Y[2,-1] + Y[1,-2])/2, (Y[-1,-2] + Y[-2,-1])/2, \\ (Y[-2,1] + Y[-1,2])/2, (Y[1,2] + Y[2,1])/2 \end{array} \right\} \quad (17)$$

$$\Theta_1 = \{Y[0,-2], Y[-2,0], Y[1,-1], Y[-1,-1], Y[-1,1], Y[1,1], Y[2,-2], Y[-2,-2]\} \quad (18)$$

$$\Theta_0 = \{Y[2,-2], Y[0,-2], Y[-2,-2], Y[-2,0]\} \quad (19)$$

where  $Y[0,0]$  is the pixel to be encoded. The integer "k" is determined by the "musk pattern" which is calculated by comparing each pixel in the musk set and mean value of the musk set. For example, for  $p=0$ ,

$$k = \sum_{r=0}^3 2^r B_r \quad (20)$$

where

$$B_r = \begin{cases} 1 & , Y_r \geq \frac{1}{4} \sum_{q=0}^3 Y_q \\ 0 & , \text{else above} \end{cases}$$

and

$$\begin{pmatrix} Y_0 & Y_1 \\ Y_2 & Y_3 \end{pmatrix} = \begin{pmatrix} Y[2,-2] & Y[0,-2] \\ Y[-2,-2] & Y[-2,0] \end{pmatrix}$$

In this case "k" does not become zero.

### 2.4 Calculation of the filter coefficients

Denoting coefficients of band "p" for context number "k" by

$$\mathbf{V}_p(k) = (a_p(k) \ b_p(k) \ c_p(k) \ d_p(k)) \quad (21)$$

we use the encoding procedure described below to renew the coefficients pixel to pixel.

**STEP 0:** Set  $p=3$  and initial value of the coefficients as

$$\begin{pmatrix} \mathbf{V}_0(l) \\ \mathbf{V}_1(l) \\ \mathbf{V}_2(l) \\ \mathbf{V}_3(l) \end{pmatrix} = \begin{pmatrix} 0.5 & 0.5 & 0 & 0 \\ 0 & 0 & 0.25 & 0.25 \\ 0.5 & 0.5 & -0.125 & -0.125 \\ 0.5 & 0.5 & -0.25 & -0.25 \end{pmatrix}$$

for  $l = 0, 1, \dots, \max(k)$

**STEP 1:** Initialize the location index  $(n_1, n_2)$  as  $(0, 0)$ .

**STEP 2:** Determine the context number "k" (does not take zero) by making the "musk pattern" from the "musk set" as described in 2.3.

**STEP 3:** Calculate a prediction error  $e_p(n_1, n_2)$  in equation (9) with the filter coefficients  $\mathbf{V}_p(k)$ .

**STEP 4:** Renew  $\mathbf{V}_p(k)$  by solving the Yule-Walker's equation for equation (15) where  $C_p = \{\text{already encoded area}\}$ . If there is no solution, assign  $\mathbf{V}_p(0)$  to  $\mathbf{V}_p(k)$ .

**STEP 5:** Increase the index  $(n_1, n_2)$  and go back to STEP2. If the index comes up to the end, then  $p=p-1$  and go to STEP1. if  $p < 0$ , then end.

### 3. SIMULATION RESULTS

We investigate effectiveness of the proposed method with the first order entropy rate [1] defined by

$$H = -\sum_i p(x_i) \log_2 p(x_i) \quad [\text{bit/pixel}]$$

where  $P(x_i)$  is the probability of a symbol  $x_i$ . We used JPEG standard still images (720x576 pixels).

**Table 1.** First order entropy rate (single stage).

method image	DPCM	EBA <sup>[8]</sup>	LMP <sup>[7]</sup>	Proposed
Barbara <sup>1</sup>	5.44	4.89	4.94	<b>4.75</b>
Boats	4.61	4.20	4.24	<b>4.00</b>
Gold-hill	4.87	4.67	4.62	<b>4.46</b>
Zelda	4.17	3.90	3.74	<b>3.59</b>
Barbara <sup>2</sup>	5.46	4.99	4.90	<b>4.79</b>
Girl	4.40	4.04	4.04	<b>3.86</b>
Hotel	4.96	4.60	4.71	<b>4.50</b>
Balloon	3.31	3.01	3.07	<b>2.84</b>
average (difference)	4.65 (0.00)	4.29 (-0.36)	4.28 (-0.37)	<b>4.10</b> (-0.55)

**Table 2.** First order entropy rate (three stages).

method image	SP <sup>[6]</sup>	FBH <sup>[13]</sup>	Proposed
Barbara <sup>1</sup>	4.89	4.93	<b>4.74</b>
Boats	4.29	4.23	<b>4.05</b>
Gold-hill	4.76	4.73	<b>4.49</b>
Zelda	3.95	3.85	<b>3.62</b>
Barbara <sup>2</sup>	5.03	4.99	<b>4.78</b>
Girl	4.09	4.00	<b>3.86</b>
Hotel	4.78	4.64	<b>4.54</b>
Balloon	3.10	2.97	<b>2.88</b>
average (difference)	4.36 (0.00)	4.29 (-0.07)	<b>4.12</b> (-0.24)

### 3.1 Single stage

As indicated in figure 2, the encoder divides an input image into four bands. Averages of the entropy rate are indicated in table 1. We attached LMP [7], EBA [8] and 2D DPCM defined by

$$y(n_1, n_2) = x(n_1, n_2) - F \left[ \frac{x(n_1 - 1, n_2) + x(n_1, n_2 - 1)}{2} \right] \quad (22)$$

The proposed method is better than EBA and LMP by 0.19 bpp and 0.18 bpp respectively in average. We can confirm effectiveness of the proposed method.

### 3.2 Multi stage

Table 2 indicates a result of "three stage octave" decomposition. The lowest band of RWT is divided into four bands again. Furthermore, resulting lowest band is divided once more. Total number of the band becomes ten. We excluded extrapolation in the lowest band, namely,  $F_{10}=0$ . SP transform [6] is indicated as "SP". A combination of S transform [12] and FIR-Boolean hybrid filter [13] is denoted as "FBH". Entropy rate is reduced by 0.24 bpp and 0.17 bpp compared to SP and FBH respectively. We can also confirm effectiveness of the proposed method.

## 4. SUMMARY

We have proposed a new "adaptive lossless scalable" coding of images. The method is based on a multi-channel non-separable

2D filter bank with adaptively changing coefficients according to local characteristics of the input image. We confirmed effectiveness of the method in respect of first order entropy rate comparing to DPCM, EBA, LMP, FBH and SP transform. Parameters of the encoder are adaptively optimized and no side information is required. Computational load should be reduced in the next step.

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