EVALUATION OF SENSITIVITY OF INTEGER DCTS

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ABSTRACT

This report compares reconstructed signal quality and confirms improvement of recently proposed six types of integer DCT algorithm such as BinDCT-IIC,-IIL, -IV, -IIS, IntDCT-II, LDCT-II [9] by means of sensitivity to finite word length. In addition, this report evaluates these integer DCT algorithms in respect of not only the sensitiveness of finite word length expression but also lossless coding performance, compatibility with the conventional DCT and computational load. It is confirmed that the LDCT-II is the best for lossless coding performance. In case of compatibility with the conventional DCT, all integer DCTs in experiment are compatible with the conventional one and the BinDCT-IIL algorithm considered by the computational load has the least number of multiplier coefficients, it is advantageous to hardware complexity.

Keywords: sensitivity, lossless, DCT, word length.

1. INTRODUCTION

The conventional DCT is widely used in many applications but not appropriate for lossless coding because of its real number output [1]. Therefore, this defect persuades many researchers to innovate this modern type of DCT to extend it to "integer" DCT that outputs integer numbers. However, it is still compatible with the conventional DCT based on lossy coding and can be utilized not only for lossless coding but also for lossy coding [2-5]. Especially, various types of the integer DCT are newly innovated or continuously improved to be better than the existing one. Optimum word length assignment is one of proposed improvement method to reduce hardware complexity and power consumption and preserve their reconstructed signal quality [6]. It refers to the theoretical analysis in term of sensitivity [7]. Soon after, the low sensitive structuring method proposed by Dang et al.[8] is proposed to find the best combination of integer DCT algorithm least sensitive to finite word length expression of its multiplier coefficient. However, it dealt with only two types of integer DCTs: Fukuma's integer DCT [2] and Soontorn's integer DCT [4] in their experiment.

This report compares quality and confirms improvement of reconstructed signal of recently proposed six types of integer DCT such as BinDCT-IIC,-IIL, -IV, -IIS, IntDCT-II, LDCT-II [9]. The quality improvement is confirmed by theoretical result in respect of uniform word length and optimum word length allocation. In addition, not only the sensitivity to finite word length but also lossless coding performance, compatibility with the conventional DCT and computational load of all algorithms are comparatively evaluated in this report.

2. INTEGER DCT ALGORITHMS

2.1 Integer Rotation Transform

Generally, integer approximation of the ordinal DCT preserves all the basic mathematical properties of the original real-valued transform matrices. For this report, we refer to the Given-Jacobi rotation matrices [9] as follows:

$$H = \begin{pmatrix} \cos\theta & -\sin\theta \\ \sin\theta & \cos\theta \end{pmatrix}, \tag{1}$$

$$X = \begin{pmatrix} x(0) & x(1) \end{pmatrix}^T, \quad Y = \begin{pmatrix} y(0) & y(1) \end{pmatrix}^T$$

where X and Y is an input vector and rotated output vector, respectively. From equation (1), it can be factorized to be the lifting matrices as follows:

Type A

$$H = \begin{pmatrix} 1 & 0 \\ c & 1 \end{pmatrix}\begin{pmatrix} 1 & b \\ 0 & 1 \end{pmatrix}\begin{pmatrix} 1 & 0 \\ a & 1 \end{pmatrix} \tag{2}$$

Type B

$$H = \begin{pmatrix} 1 & p \\ 0 & 1 \end{pmatrix}\begin{pmatrix} 1 & 0 \\ q & 1 \end{pmatrix}\begin{pmatrix} 1 & r \\ 0 & 1 \end{pmatrix} \tag{3}$$

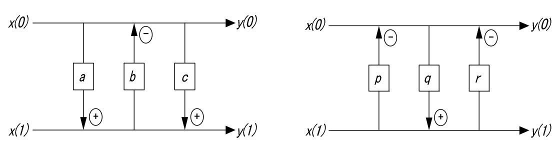
$$a = c = (1-\cos\theta)\sin^{-1}\theta, \quad b = -\sin\theta$$

$$p = r = -(1-\cos\theta)\sin^{-1}\theta, \quad q = \sin\theta$$

The "*a*", "*b*" and "*c*" are multiplier coefficients of the IR type A and the "*p*", "*q*" and "*r*" are multiplier coefficients of the IR type B illustrated in figure 1. All the integer DCT types include different number of IRs and each IR includes three multiplier coefficients. Normally, for lossy coding, each multiplier coefficient is real number identical to the conventional DCT. On the contrary, for lossless coding, the word length of each multiplier coefficient is truncated to be finite word length.

2.2 Integer DCT Algorithms

The low sensitive structuring method proposed by Dang et al. [8] is applied to six types of recently proposed integer DCTs shown in Fig. 2 to 7 such as BinDCT-IIC,-IIL, -IV, -IIS, IntDCT-II, LDCT-II [9] for comparison. These figures show the signal flow graph for the forward transform of each integer DCT. A dotted line denotes negative sign. Each of the integer DCTs contains the integer rotation (IR) transform shown in figure 1. The permutation (P) does not contain any multiplier coefficients but just only rearrange input or output of transform algorithm.



(a) Type A (b) Type B

Fig. 1 Two types of integer rotation (IR) transforms.

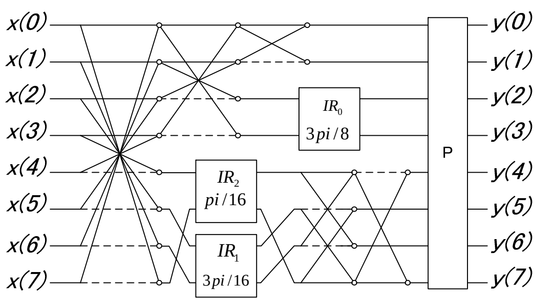


Fig. 2 BinDCT-IIL algorithm

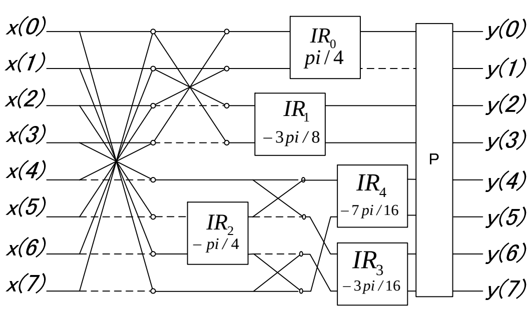


Fig.3 BinDCT-IIC algorithm

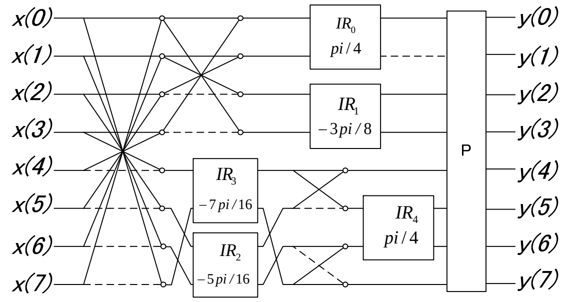


Fig.4 BinDCT-IIS algorithm

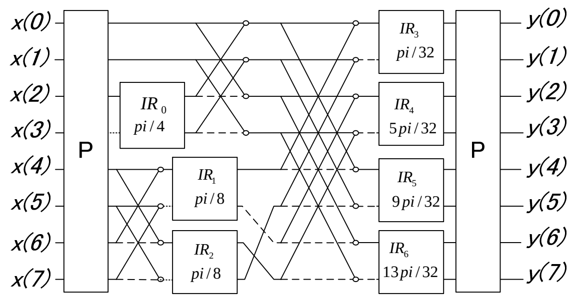


Fig. 5 BinDCT-IV algorithm

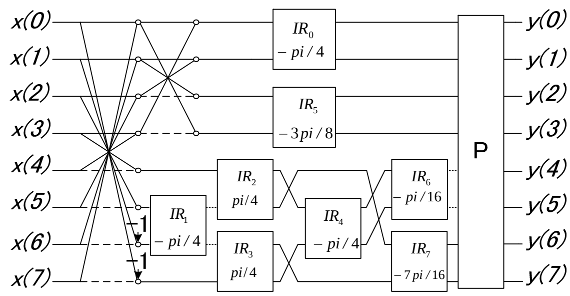


Fig.6 LDCT-II algorithm

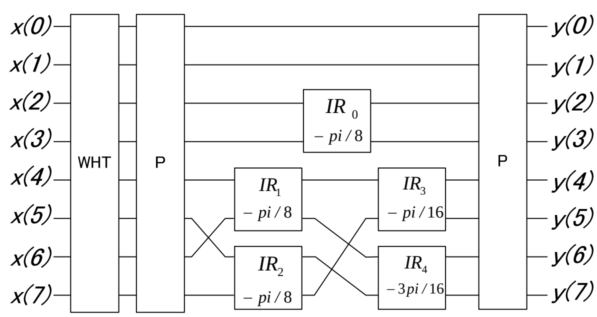


Fig.7 IntDCT-II algorithm

3. WORD LENGTH ASSIGNMENT

3.1 Evaluation of the Error

Referring to the theoretical investigation of dependence of the word length assignment on different algorithms [7], we can find the quality of reconstructed signal given by

$$PSNR = 10\,log_{10}\frac{255^2}{\sigma^2_{\Delta total}} \qquad (4)$$

where $\sigma^2_{\Delta total}$ denotes the variance of total error. It is calculated by

$$\sigma^2_{\Delta total} = \sum_{k=0}^{K-1}\sigma^2_{\Delta y_k} \qquad (5)$$

Since $\sigma^2_{\Delta y_k} = S_k^2 \Delta h_k^2$ and it denotes the variance of output error due to truncated multiplier coefficient h_k, the variance of total error is

$$\sigma^2_{\Delta total} = \sum_{k=1}^{K} S_k^2 . \Delta h_k^2 \qquad (6)$$

where Δh_k denotes the truncation error of multiplier coefficient h_k. The probability of Δh_k is a uniform histogram in range of $0 \le \Delta h_k \le 2^{-(W_k+1)}$, so the worst case of PSNR is considered by the maximum range at $\Delta h_k = 2^{-(W_k+1)}$. Then,

$$\sigma^2_{\Delta total} = \sum_{k=1}^{K} S_k^2 . 2^{-2(W_k+1)} \qquad (7)$$

Substituting Eq.(7) into Eq.(4) as

$$PSNR = 10\,log_{10} 255^2 - 10\,log_{10} \sum_{k=1}^{K} S_k^2 . 2^{-2(W_k+1)} \qquad (8)$$

The comparative quality improvement of each integer DCT can be evaluated by the difference of PSNR between the proposed and existing integer DCT algorithm by the following equation.

$$\Delta PSNR = PSNR_{proposed} - PSNR_{existing} \qquad (9)$$

3.2 Uniform word length allocation

In case of a uniform word length allocation, we assign the same word length of all multiplier coefficients. The quality of reconstructed signal in worst case is defined by Eq.(8) and the comparative quality improvement is the difference of PSNR between the proposed and existing integer DCT algorithm using Eq.(9) by

$$\Delta PSNR_{uniform} = -10\,log\frac{\sum_{k=1}^{K} S^2_{k(proposed)}}{\sum_{k=1}^{K} S^2_{k(existing)}} \qquad (10)$$

From Eq. (10), it is found that the quality improvement of reconstructed signal for the uniform word length allocation is a function of summation of sensitivity of existing and proposed integer DCT algorithm.

3.3 Optimum word length allocation

Referring to the optimum word length assignment [6], this method is proved by minimizing total energy $\sigma^2_{total} = \sum_{k=1}^{K}\sigma_k^2$ under a given average word length $\overline{W}$ and solved by linear equations with the Lagrange's method [6]. Word length of each multiplier can be obtained by

$$W_k = \overline{W} + log_2\frac{S_k}{\overline{S}}, \qquad k \in \{0,1,\cdots,K-1\} \qquad (11)$$

where

$$\overline{W} = \frac{1}{K}\sum_{k=0}^{K-1} W_k \qquad (12)$$

W_k and $\overline{W}$ denote word length of each multiplier coefficient h_k and average word length respectively. The PSNR improvement in case of the optimum word length assignment is defined by substituting Eq.(11) into Eq.(8). Therefore, we can find the comparative quality improvement using Eq.(9) in term of average sensitivity as follows

$$\Delta PSNR_{optimizing} = -10\,log\frac{\overline{S}^2_{(proposed)}}{\overline{S}^2_{(existing)}} \qquad (13)$$

4. PERFORMANCE EVALUATION

4.1 Sensitivity

The low sensitive structuring method proposed by Dang et al. [8] is utilized to find the best combination in respect of the least sensitivity as the proposed algorithm comparing with the existing one in which it includes only type A of IRT. The AR (1) model with $\rho = 0.95$ is used as a colored input signal and average word length $\overline{W}$=4 bits for this experiment. As a result, the methodical [8] and all type A algorithms of six types of integer DCT are shown in Table 1. Our experiment focuses on the comparative quality improvement of reconstructed signal of these methodical [8] and all type A integer DCTs in which both algorithms are assigned the uniform word length allocation and the optimum word length allocation method to their multiplier coefficients.

The comparative quality improvement of reconstructed signal of each integer DCT is theoretically evaluated between the methodical [8] and all type A algorithm by using Eq. (10) and Eq. (13) is summarized in table 2. While these algorithms are assigned the optimum word length to each multiplier coefficients, it is found that the PSNR improvement of BinDCT-IV is up to 2.286 dB and that of IntDCT-II is up to 2.258 dB. On the contrary, in case of the uniform word length allocation, the PSNR improvement is low due to the effect of sensitivity to finite word length.

Comparison of experimental and theoretical result between all type A and method [8] of each integer DCT algorithm is illustrated in Fig. 8 to 11. Figure 8 and 9 are the sample of the PSNR improvement by the uniform word length allocation of BinDCT-IV and IntDCT-II respectively. Figure 10 and 11 are the sample of the PSNR improvement by the optimum word length allocation of BinDCT-IV and IntDCT-II respectively. It is found that the PSNR is evidently improved when we assign the optimum word length to each multiplier coefficient of integer DCT.

Table 1 All type A and method [8] of each integer DCT algorithm

	BinDCT-IIL		BinDCT-IIC		BinDCT-IIS		BinDCT-IV		LDCT-II		IntDCT-II	
	All type A	Method [8]	All type A	Method [8]	All type A	Method [8]	All type A	Method [8]	All type A	Method [8]	All type A	Method [8]
IR0	A	A	A	B	A	B	A	B	A	B	A	B
IR1	A	B	A	B	A	B	A	B	A	B	A	B
IR2	A	A	A	A	A	B	A	B	A	A	A	B
IR3	-	-	A	A	A	A	A	B	A	A	A	B
IR4	-	-	A	B	A	A	A	B	A	B	A	B
IR5	-	-	-	-	-	-	A	A	A	B	A	B
IR6	-	-	-	-	-	-	A	B	A	B	-	-
IR7	-	-	-	-	-	-	-	-	A	A	-	-

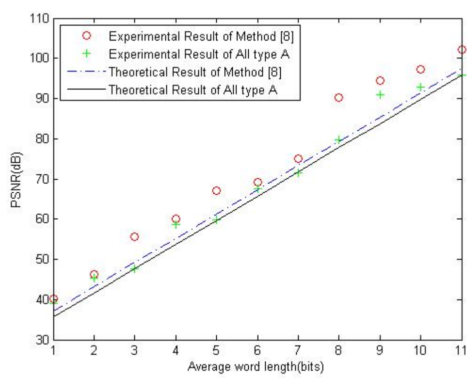


Fig.8 PSNR improvement of BinDCT-IV by uniform word length allocation.

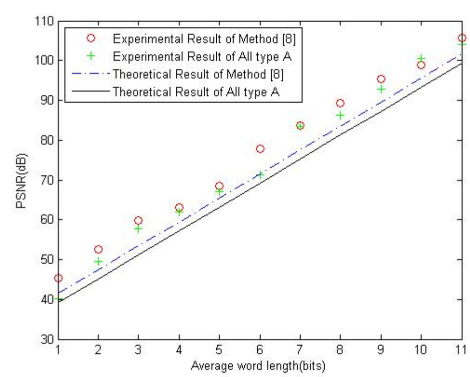


Fig.10 PSNR improvement of BinDCT-IV by optimum word length allocation.

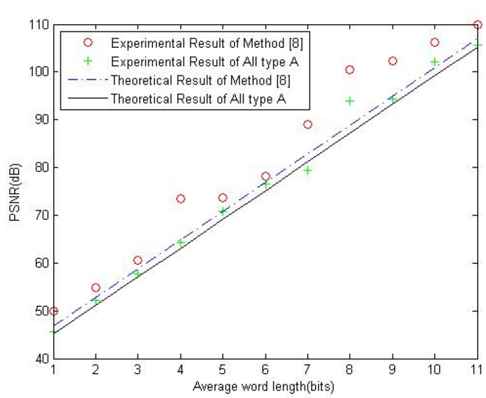


Fig.9 PSNR improvement of IntDCT-II by uniform word length allocation.

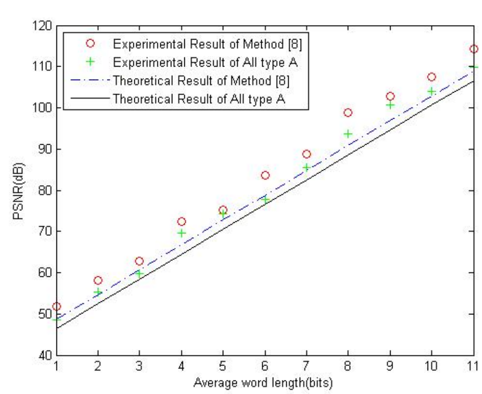


Fig.11 PSNR improvement of IntDCT-II by optimum word length allocation.

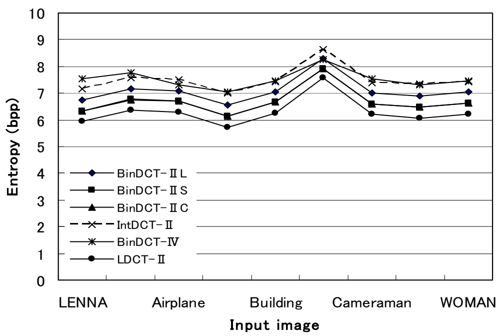


Fig.12 Total entropy rate (bpp) for lossless coding.

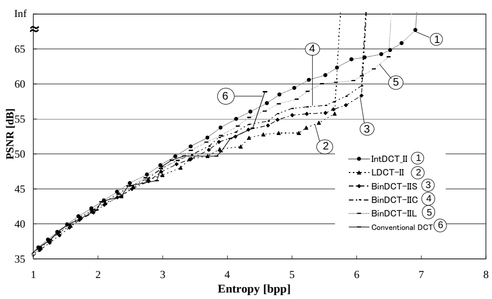


Fig.13 Rate distortion curve of a decoded image "Lenna".

Table 2 Comparative PSNR improvement.

Type	PSNR Improvement [dB]	
	Uniform Word length Allocation	Optimum Word length Allocation
BinDCT-IIL	-0.213	-0.028
BinDCT-IIC	0.039	1.107
BinDCT-IIS	0.01	1.616
BinDCT-IV	1.552	2.286
LDCT-II	0.045	0.851
IntDCT-II	1.705	2.258

Table 3 The number of IRs and multipliers

Type	The number of IRs	The number of multipliers
BinDCT-IIL	3	9
BinDCT-IIC	5	15
BinDCT-IIS	5	15
BinDCT-IV	7	21
LDCT-II	8	24
IntDCT-II	5	15

4.2 Lossless Coding

Comparative performance in aspect of lossless coding is shown in Fig.12. Some standard images are used as input signal. It is found that the first order entropy rate of LDCT-II is the best and that of BinDCT-IV and IntDCT-II is the worst. A line of entropy rate in case of BinDCT-IV is not compatible to integer DCT type II so its shape is not similar to the others.

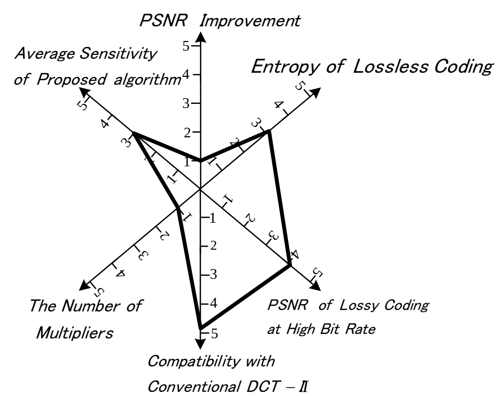
4.3 Compatibility with the conventional DCT

Figure 13 shows the compatibility of integer DCTs type II with the conventional DCT type II in respect of rate distortion curve. While we apply the conventional lossy DCT that all of its multiplier coefficients are real number, it is found that the PSNR of each integer DCT is nearly similar to the case of the conventional DCT at low bit rate but it is not compatible at high bit rate because of rounding effect.

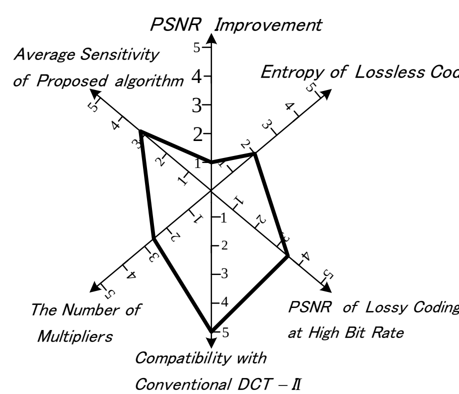
4.4 Computational Load

For hardware implementation, computational load is considered by the number of multiplier coefficients of each integer DCT illustrated in table 3. It is found that the BinDCT-IIL has the least number of multiplier coefficients in which it is advantageous to hardware complexity. Meanwhile, the LDCT-II has the most complexity because of the most number of multipliers.

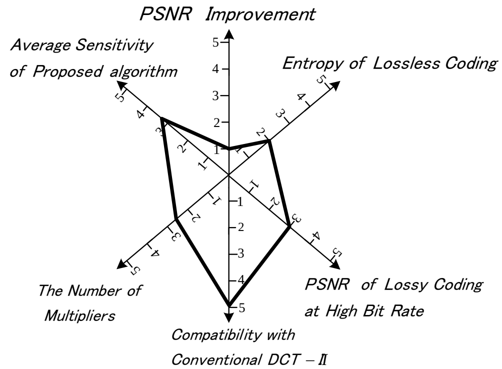
All the results of comparing the six integers DCT algorithms are confirmed by the normalized scale of each criterion in Fig. 14.



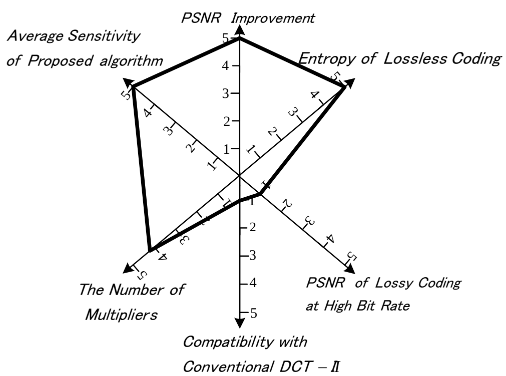
(a) BinDCT-IIL



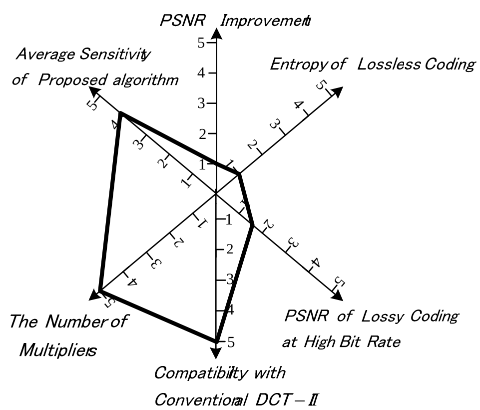
(b) BinDCT-IIC



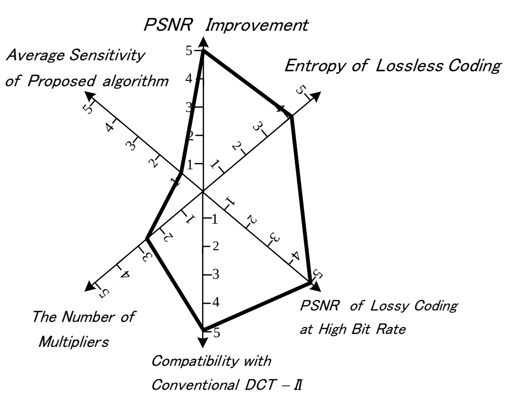
(c) BinDCT-IIS



(d) BinDCT-IV



(e) LDCT-II



(f) IntDCT-II

Fig. 14 Results of comparing the six integer DCTs.

5. CONCLUSION

All integer DCT algorithms were comparatively evaluated. It is found that PSNR improvement depends on sensitivity to finite word length. The highest PSNR improvement of BinDCT-IV is up to 2.286 dB by optimum word length allocation. For lossless coding performance, The LDCT-II algorithm uses the minimum bit rate. In case of compatibility with the conventional DCT, all integer DCTs are compatible with the conventional one at low bit rate but they are not at high bit rate. The PSNR of IntDCT-II is the highest at high bit rate for lossy coding. For hardware complexity, it is found that the BinDCT-IIL algorithm has the least number of multiplier coefficients, so it is more advantageous to hardware implementation.

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