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AVOIDANCE OF SINGULAR POINT IN REVERSIBLE KLT

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ABSTRACT

In this report, permutation of order and sign of signals are introduced to avoid singular point problem of a reversible transform. When a transform is implemented in the lifting structure, it can be "reversible" in spite of rounding operations inside the transform. Therefore it has been applied to lossless coding of digital signals. However some coefficient values of the transform have singular points (SP). Around the SP, rounding errors are magnified to huge amount and the coding efficiency is decreased. In this report, we analyze the SP of a three point KLT for RGB color components of an image signal, and introduce permutation of order and sign of signals to avoid the SP problem. It was experimentally confirmed that the proposed method improved PSNR by approximately 15 [dB] comparing to the worst case.

Index Terms— error, KLT, reversible, coding, color

1. INTRODUCTION

Over the past few decades, a considerable number of studies have been made on orthonormal transforms such as DCT and KLT for lossy coding of audio or image signals [1-4]. Ever since the introduction of the lifting structure, lossless coding by a reversible transform has been also brought to light [5-7]. So far, a great deal of effort has been made on constructing reversible transforms [2]. What seems to be lacking, however, is the singular point (SP) problem peculiar to the lifting structure.

When a transform is implemented in the lifting structure, it can be "reversible" in spite of non linear operations such as rounding of signal values and coefficient values inside the transform circuit. Benefitting from this property, the reversible transform has been applied to lossless coding of digital signals. However, in this case, some coefficient values of the transform have singular points (SP) [8]. Around the SP, rounding errors are magnified to huge amount and the coding efficiency is decreased.

In this report, we analyze the SP of a three point KLT which is applied to RGB color components of an image signal. To avoid the SP problem, we introduce permutation of order and sign of signals. Analyzing effect of these permutations on rotation angles, magnification problem of the rounding error by the SP is avoided. We also reduce the time complexity of the optimization procedure from exponential time of the existing approach in [9,10] to polynomial time.

The SP problem and its avoidance in the existing method are briefly addressed in **2.** The proposed method is described in **3.** Effectiveness of the proposed method is evaluated in **4.** This report is summarized in **5.**.

2. REVERSIBLE KLT AND ITS SINGULAR POINT

2.1. Factorization of KLT into Two-Point Matrices

The three-point KLT converts s a set of input values $\mathbf{X}$ into a set of output values $\mathbf{Y}$ as

$$\mathbf{Y} = \mathbf{K}^T \mathbf{X} \tag{1}$$

where

$$\mathbf{Y} = [y_i \quad y_j \quad y_k]^T, \quad \mathbf{X} = [x_i \quad x_j \quad x_k]^T$$

with the transform matrix $\mathbf{K}$. In this report, $\mathbf{X}$ is a set of RGB color components. The matrix $\mathbf{K}$ is determined as eigenvectors of the covariance matrix $\mathbf{R_X}$ defined as

$$\mathbf{R_X} = E[\mathbf{X}\mathbf{X}^T] \tag{2}$$

and it converts $\mathbf{R_X}$ into

$$\begin{aligned}\mathbf{R_Y} &= E[\mathbf{Y}\mathbf{Y}^T] = \mathbf{K}^T E[\mathbf{X}\mathbf{X}^T]\mathbf{K} = \mathbf{K}^T \mathbf{R_X} \mathbf{K} \\ &= diag[\lambda_i \quad \lambda_j \quad \lambda_k]\end{aligned} \tag{3}$$

to decorrelate the components.

The 3×3 matrix $\mathbf{K}$ in Eq.(1) can be factorized into a product of 2×2 matrices:

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$$\mathbf{G}(\varphi)=\begin{bmatrix}\cos\varphi & -\sin\varphi\\ \sin\varphi & \cos\varphi\end{bmatrix},\ \mathbf{H}(\varphi)=\begin{bmatrix}\cos\varphi & \sin\varphi\\ \sin\varphi & -\cos\varphi\end{bmatrix} \quad (4)$$

where $\mathbf{G}(\theta)$ and $\mathbf{H}(\theta)$ denote Givens rotation and Householder reflection, respectively. Fig.1(a) illustrates an example of the factorization of **K**. It should be noted that a set of rotation angles $\{\varphi_i, \varphi_j, \varphi_k\}$ is determined by the covariance matrix $\mathbf{R_X}$. Namely, the KLT matrix **K** varies in accordance with statistical characteristics of an input color image signal.

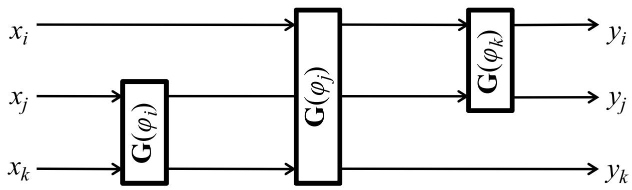


Fig.1 A three-point KLT matrix **K** is factorized into two-point rotation matrices **G**.

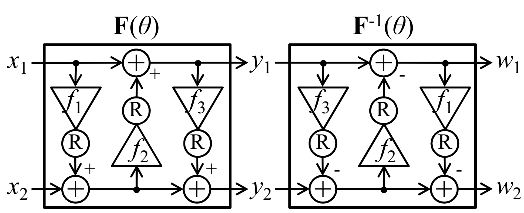


Fig.2 A pair of reversible two-point rotation transforms. Forward transform **F** and backward transform $\mathbf{F}^{-1}$.

2.3. Singular Point Problem

Fig.2 illustrates a pair of reversible transforms. Their output signals are calculated as

$$\begin{bmatrix}x'_2\\ y_1\\ y_2\end{bmatrix}=\begin{bmatrix}x_2+R[f_1x_1]\\ x_1+R[f_2x'_2]\\ x'_2+R[f_3y_1]\end{bmatrix},\ \begin{bmatrix}y'_2\\ w_1\\ w_2\end{bmatrix}=\begin{bmatrix}y_2-R[f_3y_1]\\ y_1-R[f_2y'_2]\\ y'_2-R[f_1w_1]\end{bmatrix} \quad (5)$$

where $R[x]$ denotes rounding of x to the nearest integer. In this lifting structure, its output $\{w_1, w_2\}$ is exactly the same as its integer input $\{x_1, x_2\}$ in spite of the rounding operations since the rounding errors are totally cancelled between input of the forward transform $\mathbf{F}(\theta)$ and output of the backward transform $\mathbf{F}^{-1}(\theta)$.

In case of the rounding errors are negligible, Eq.(5) is described as

$$\begin{bmatrix}y_1\\ y_2\end{bmatrix}=\begin{bmatrix}1 & 0\\ f_3 & 1\end{bmatrix}\begin{bmatrix}1 & f_2\\ 0 & 1\end{bmatrix}\begin{bmatrix}1 & 0\\ f_1 & 1\end{bmatrix}\begin{bmatrix}x_1\\ x_2\end{bmatrix}=\mathbf{F}(\theta)\begin{bmatrix}x_1\\ x_2\end{bmatrix} \quad (6)$$

where $\{f_1, f_2, f_3\}$ are multiplier coefficients. When all the rotations **G** in Fig.1 are replaced by the reversible transform in Fig.2, the KLT becomes "reversible". In this case, comparing Eq.(4) and Eq.(6), the equation $\mathbf{F}(\theta) = \mathbf{G}(\varphi)$ implies

$$f_1=f_3=\tan\frac{\varphi}{2},\qquad f_2=-\sin\varphi\ . \quad (7)$$

The equation above indicates that absolute value of f_1 and f_3 are close to infinity as φ is close to π [rad]. This is the singular point problem we investigate in this report.

In implementation of the reversible KLT, signal values and coefficient values are rounded to be expressed with finite word length. It is inevitable to have rounding errors in the transformed values $\{y_1, y_2\}$ in Fig.2 and $\{y_i, y_j, y_k\}$ in Fig.1. Total amount of these errors are measured with PSNR and reduced by the methods explained in the following sections.

2.4. Existing Method

Fig.3(a) illustrates an existing approach reported in [9,10]. The SP problem of the reversible KLT is avoided by introducing the permutation matrices $\mathbf{Q}_a$ and $\mathbf{Q}_b$. Each of them is given as a product of permutation of sign $\mathbf{S}_3$ and permutation of order $\mathbf{P}_3$, namely,

$$\mathbf{Q}_a, \mathbf{Q}_b \in \mathbf{S}_3\mathbf{P}_3 \quad (8)$$

where

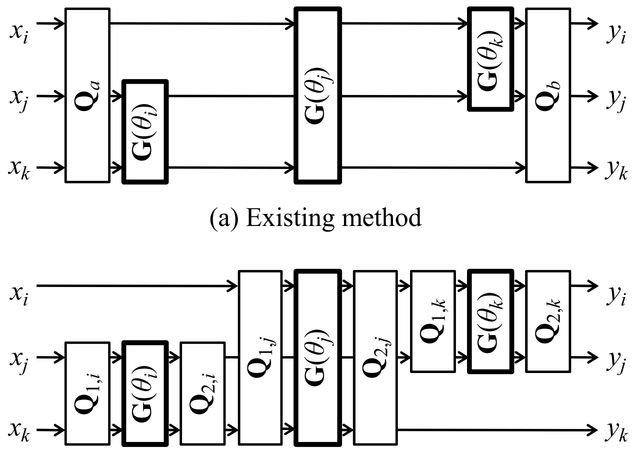
$$\mathbf{S}_3=\begin{bmatrix}(-1)^a & 0 & 0\\ 0 & (-1)^b & 0\\ 0 & 0 & (-1)^c\end{bmatrix},\quad a,b,c\in\{0,1\} \quad (9)$$

and

$$\mathbf{P}_3\in\left\{\begin{bmatrix}1&0&0\\0&1&0\\0&0&1\end{bmatrix},\begin{bmatrix}0&1&0\\1&0&0\\0&0&1\end{bmatrix},\begin{bmatrix}1&0&0\\0&0&1\\0&1&0\end{bmatrix},\begin{bmatrix}0&1&0\\0&0&1\\1&0&0\end{bmatrix},\begin{bmatrix}0&0&1\\1&0&0\\0&1&0\end{bmatrix},\begin{bmatrix}0&0&1\\0&1&0\\1&0&0\end{bmatrix}\right\}. \quad (10)$$

In this existing method, the rotation angles $\{\theta_i, \theta_j, \theta_k\}$ vary in accordance with selection of the permutation matrices $\mathbf{S}_3$ and $\mathbf{P}_3$ for a given KLT matrix **K** for an input color image signal. Since $\mathbf{S}_3$ and $\mathbf{P}_3$ have 2^3 and 3! candidates respectively, the existing method selects the best one from all the $(2^3\,3!)^2 = 2{,}304$ combinations.

It should be noted that the existing method requires exponential time for selection of the best of all the combinations to avoid the SP problem.



(a) Existing method

(b) Proposed method

Fig.3 The singular point problem is avoided by introducing the permutation matrices **Q**.

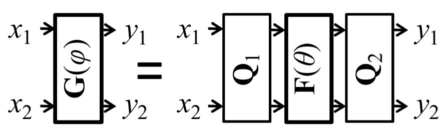


Fig.4 A reversible rotation transform with permutations.

3. PROPOSED METHOD

3.1. Introduction of Permutations

Fig.3(b) illustrates the proposed method. We introduce the permutation matrices $\mathbf{Q}_{p,q}$, $p \in \{1, 2\}$, $q \in \{i, j, k\}$ as a product of permutation of sign $\mathbf{S}_2$ and permutation of order $\mathbf{P}_2$ where

$$\mathbf{P}_2 = \begin{bmatrix} 0 & 1 \\ 1 & 0 \end{bmatrix}, \quad \mathbf{S}_2 = \begin{bmatrix} -1 & 0 \\ 0 & 1 \end{bmatrix}, \quad \mathbf{I}_2 = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}. \quad (11)$$

Namely, a rotation transform $\mathbf{G}(\varphi)$ is implemented as the reversible transform $\mathbf{F}(\theta)$ with permutations $\mathbf{Q}_1$ and $\mathbf{Q}_2$ as illustrated in Fig.4. Since these matrices have the property:

$$\mathbf{P}_2^2 = \mathbf{I}_2, \quad \mathbf{S}_2^2 = \mathbf{I}_2, \quad (\mathbf{P}_2\mathbf{S}_2)^2 = (-\mathbf{S}_2\mathbf{P}_2)^2 = -\mathbf{I}_2, \quad (12)$$

and $\mathbf{P}_2$=$\mathbf{H}(\pi/2)$, $\mathbf{S}_2$=$\mathbf{H}(\pi)$, $\mathbf{I}_2$=$\mathbf{G}(0)$, the permutation in our method has a structure of the dihedral group of order 8. As a result, one of eight candidates illustrated in Fig.5 is selected as the permutation $\mathbf{Q}_1$ and $\mathbf{Q}_2$ in Fig.4.

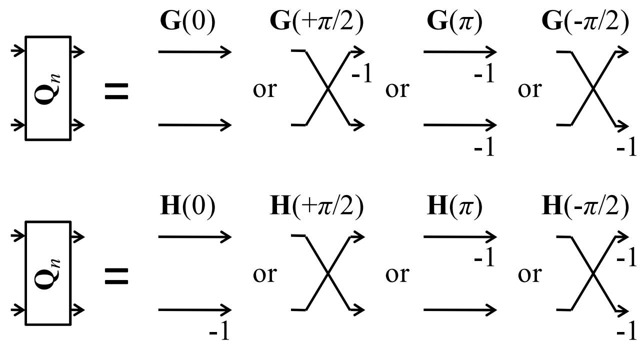


Fig.5 One of Givens rotations and Householder reflections in the figure is selected as a permutation.

3.2. Effect of Permutation on Rotation Angles

Utilizing the property of the Givens rotation **G** and the Householder reflection **H** :

$$\begin{cases} \mathbf{G}(\beta)\mathbf{G}(\alpha) = \mathbf{G}(\beta+\alpha) \\ \mathbf{H}(\beta)\mathbf{G}(\alpha) = \mathbf{H}(\beta-\alpha) \\ \mathbf{G}(\beta)\mathbf{H}(\alpha) = \mathbf{H}(\beta+\alpha) \\ \mathbf{H}(\beta)\mathbf{H}(\alpha) = \mathbf{G}(\beta-\alpha) \end{cases} \quad (13)$$

we select one of four candidates given as

$$\begin{cases} case(1) & \mathbf{G}(\varphi) = \mathbf{G}(0)\mathbf{F}(\theta)\mathbf{G}(0) \\ case(2) & \mathbf{G}(\varphi) = \mathbf{G}(0)\mathbf{F}(\theta)\mathbf{G}(+\pi/2) \\ case(3) & \mathbf{G}(\varphi) = \mathbf{G}(0)\mathbf{F}(\theta)\mathbf{G}(\pi) \\ case(4) & \mathbf{G}(\varphi) = \mathbf{G}(0)\mathbf{F}(\theta)\mathbf{G}(-\pi/2) \end{cases} \quad (14)$$

for each reversible rotation transform $\mathbf{G}(\varphi_q)$, $q \in \{i, j, k\}$ in Fig.1. Namely, in the "full search" of the proposed method, the best one of all the (4^3) = 64 combinations is determined.

Furthermore, since relations between the angles are given as

$$\begin{cases} case(1) & \theta = \varphi \\ case(2) & \theta = \varphi - \pi/2 \\ case(3) & \theta = \varphi - \pi \\ case(4) & \theta = \varphi + \pi/2 \end{cases} \quad (15)$$

for $\mathbf{F}(\theta)$=$\mathbf{G}(\theta)$ in Eq.(14), we can select the best one so that distance between the SP and the angle θ becomes maximum. In this "distance" based search case, the proposed method selects the best one from only (4×3) = 12 combinations.

It should be noted that the proposed method requires polynomial time for selection of the best one to avoid the SP problem.

4. EXPERIMENTAL RESULTS

Fig.6 compares the proposed method to the existing method with total amount of the rounding errors in output signals of the forward transform. In the figure, peak signal to noise ratio (PSNR) is used as a measure. When the best combination is selected, both of the two methods attain approximately 54 [dB]. It is worth paying attention to the fact that PSNR could be dropped to approximately 39 [dB] in the worst case.

Fig.7 compares two search methods in the proposed method. "Full search" and "distance" based search determine the best one from 4^3 and 4×3 combinations respectively. The difference was found to be only 0.1 [dB] at maximum.

In our experiments, it was found that there was no significant difference between the proposed method and the existing method in respect of total amount of the rounding error. However, the time complexity of the selection procedure was dramatically reduced from $(2^3\ 3!)^2$ times variance calculations to only 4×3 times distance calculations.

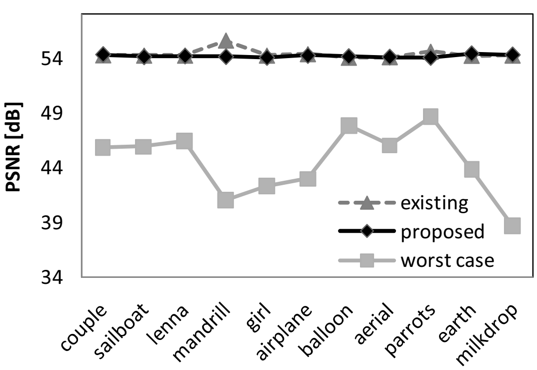


Fig.6 Total amount of the rounding errors.

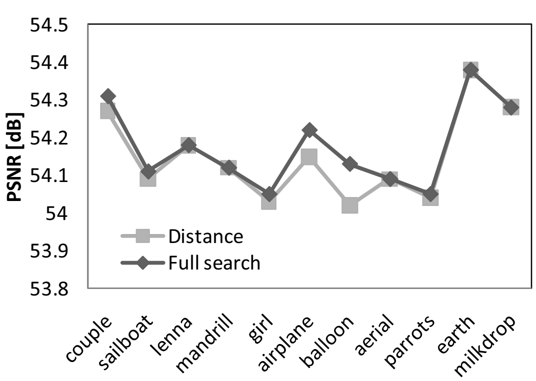


Fig.7 Two searches in the proposed method.

5. CONCLUSIONS

In this report, we focused on the singular point (SP) of a three point KLT for RGB color components of an image signal. We introduced permutation of order and sign of signals to avoid the SP problem. Analyzing effect of these permutations on rotation angles, magnification of the rounding error by the SP was avoided. It was confirmed that the proposed method improved PSNR by approximately 15 [dB] comparing to the worst case. The time complexity of the selection procedure is reduced from exponential time to polynomial time.

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