Correspondence

A Development of Symmetric Extension Method for Subband Image Coding

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***Abstract*—This correspondence describes a development of a technique for subband image coding called symmetric extension method which utilizes the nature of symmetrically extended image to achieve high quality coding. There are, however, some restrictions on the usable subband coding system. The development is done based on the property of symmetrically extended signal in analysis process to remove the restriction. The behavior of the sequence after two processes, filtering and decimation, is examined and formulated. Our development extends the application area of the technique.**

I. INTRODUCTION

This paper deals with a problem which will occur when Subband Coding [1] is applied to image coding, i.e. the problem of increasing image size due to the finiteness of image data. As shown in Fig. 1, a subband image coding system consists of filter banks and a coding system. In this system, the problem occurs after the analysis process. That is, the sum of the pixels of subimages $y_{nm}[m_1, m_2]$ become larger than that of input $x[n_1, n_2]$.

So far, many researchers have proposed several methods for solving this problem. Woods and O'Neil suggested using circular convolution instead of linear convolution [1]. Smith *et al.* showed that this method can be implemented by expanding the input image periodically and they named it the circular convolution method [2]–[4]. This method requires concatenating one edge of the finite-length signal with the other which generates very sharp transition and filtering must be done on this transition. This leads to visible distortion at the edges of the reconstructed image.

In their papers, Smith *et al.* proposed another efficient method called symmetric extension [2]–[4]. This method uses the symmetric extension of the original sequence $x[n]$, which is called $x_{se}[n]$

$$x_{se}[n] = \begin{cases} x[n], & 0 \geq n \geq N-1 \\ x[2N-n-1], & N \geq n \geq 2N-1 \\ 0, & \text{otherwise} \end{cases} \qquad (1)$$

as an input sequence to the circular convolution method. In Fig. 2, we show the two methods of extending a sequence. When the original sequence $x[n]$ is given as (a), then we obtain (b) by periodically extending (a) and we obtain (c) by symmetric extension of (a). Since symmetrically extended sequences have a smoother boundary than that of periodically extended sequences the symmetric extension method provides a better solution than the circular convolution method. However, there are restrictions on the filter type and on the structure of the filter bank. Namely, filters must have linear phase, the number of their impulse responses must be even, and the filter bank structure must be a tree structure of two-band filter banks.

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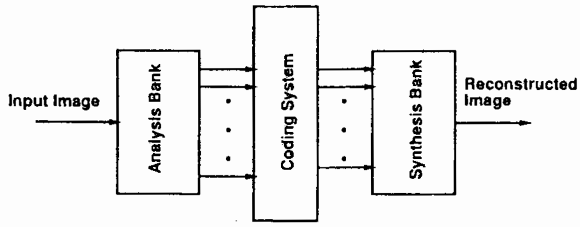

Fig. 1. Typical subband image coding system.

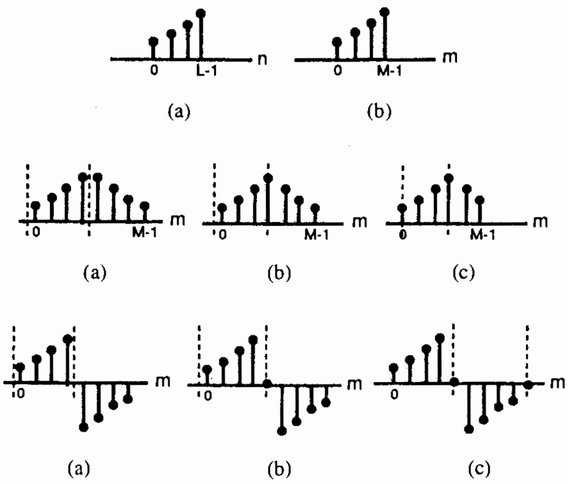

Fig. 2. Various methods to generate periodic sequences. (Broken line indicates a center of symmetry in a period.) (a) Nonperiodic sequence (length L). (b) Asymmetric periodic sequence (M = L). (c) Case A (M = 2L) (half-sample symmetry). (d) Case B (M = 2L − 1). (e) Case C (M = 2L − 2) (whole-sample symmetry). (f) Case D (M = 2L). (g) Case E (M = 2L + 1). (h) Case F (M = 2L + 2).

Development of the symmetric extension method is presented by Martucci [5]. By researching the property of the perfect reconstruction system, he derived the conditions under which the method works. He showed there exists two ways of extending a sequence including Smith's method. He named the two methods whole-sample symmetry and half-sample symmetry, where half-sample symmetry is given to Smith's method. The half-sample symmetry sequence's point of symmetry lies at the half-way point between two samples of the sequence and that of the whole-sample symmetry as it lies on one of the samples of the sequence. In Fig. 2, the half-sample symmetry sequence is shown in (c) and the whole-sample symmetry sequence in (e). Using these two types of symmetry, any type of linear phase FIR filters can be used as the filters which consist of filter banks. Even after Martucci's development, there still exist restrictions on the number of division since only two-band filter bank cases had been developed.

This paper presents further development of the symmetric extension method. We are considering the properties of the symmetric sequence. We propose six methods for generating symmetric sequence including the previous two methods. We formulate the symmetric sequences using frequency domain representation and use this formula to investigate the behavior of a sequence in the analysis process. Our development enables us to apply the symmetric extension method to the system of any number of divisions.

We use the 1-D notation in the following consideration under the assumption that 2-D filtering will be implemented as a set of 1-D filtering operations. Filtering is first performed on each row and then on each column of the image. Therefore, the theory will be developed in terms of 1-D sequences and systems.

II. The Symmetrically Extended Sequence

Different from Martucci's approach, we use the properties of the symmetrical sequence to develop the symmetric extension method. The effect of a decimating process on a symmetrical sequence is considered. How the symmetric extension method works is shown.

A. The Formulation

We can generate six types of symmetrical sequences from the original sequence [6], half of which are symmetric and the other half are antisymmetric. We refer to the three types of symmetric sequences as cases A, B, and C, respectively, and the three antisymmetric as cases D, E, and F and they are shown in Fig. 2 (c)–(h). Note that case A sequence is referred to as half-sample symmetry and case C sequence as whole-sample symmetry in [5]. However, case B, which is an odd length one, has never been mentioned before because it has no role in the two-band filter banks based system. This case B sequence enables us to use the odd-band system.

Let us represent these symmetric sequences using discrete Fourier series (DFS). The DFS of a periodical sequence $\tilde{x}[n]$ is defined as

$$\tilde{X}[k] = \sum_{n=0}^{M-1} \tilde{x}[n] W_M^{kn} \quad (2)$$

where M denotes the period of $\tilde{x}[n]$, $W_M = \exp(-j2\pi/M)$, and $\tilde{}$ (tilde) is used to indicate the periodic sequence.

Using (2), the six types of symmetrical sequences can be expressed as below.

$$\tilde{X}_A[k] = W_M^{ks} W_M^{k(M-1)/2} \tilde{X}_A^*[k] \quad M\text{:even} \quad (3a)$$
$$\tilde{X}_B[k] = W_M^{ks} W_M^{k(M-1)/2} \tilde{X}_B^*[k] \quad M\text{:odd} \quad (3b)$$
$$\tilde{X}_C[k] = W_M^{ks} W_M^{kM/2} \tilde{X}_C^*[k] \quad M\text{:even} \quad (3c)$$
$$\tilde{X}_D[k] = j\tilde{X}_A[k] \quad (3d)$$
$$\tilde{X}_E[k] = j\tilde{X}_B[k] \quad (3e)$$
$$\tilde{X}_F[k] = j\tilde{X}_C[k] \quad (3f)$$

where $j = \sqrt{-1}$, $\tilde{X}_\alpha^*[k]$ is purely real and the subscript letter represents the type of periodic sequence. In these equations s is an arbitrary integer representing the shift value. Obviously from these equations, the sequences of case D to F can be thought as having the same property of the sequences of case A to C except they are multiplied by j. This means we can know the property of the symmetrically extended sequence only by reserching the peroperies of symmetric sequences, i.e., cases A to C.

In general, we can express these equations in a unified form, namely

$$\tilde{X}_\alpha[k] = e^{jT} W_M^{kL} \tilde{X}_\alpha^*[k]. \quad (4)$$

In this equation, the type of a symmetrically extended sequence is determined by three parameters, i.e., M, L and T. In Table I, we tabulate them for each type of periodic sequence.

TABLE I
Values of Three Parameters M, L, and T for Each Type of Symmetrically Extended Sequence

Sequence	M	L	T
A	even	Half Int.	$2n\pi$
B	odd	Int.	$2n\pi$
C	even	Int.	$2n\pi$
D	even	Half Int.	$2n\pi + \pi/2$
E	odd	Int.	$2n\pi + \pi/2$
F	even	Int.	$2n\pi + \pi/2$

n : integer
Int. : integer

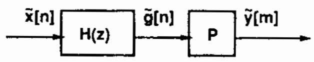

Fig. 3. Part of the analysis process.

III. Influence of Analysis Process

In this section, we study the properties of a symmetrically extended sequence in the analysis process. As shown in Fig. 3, the analysis process is executed using two components, namely a filter and a decimator. We study their respective influences on a symmetric sequence and show that the next two conditions must be satisfied for the symmetric extension methods to work.

1) The period M of $\tilde{x}[n]$ must be an integer multiple of P. Because the period of $\tilde{y}[m]$ is M/P and it must be an integer.
2) $\tilde{y}[m]$ must be a symmetrical sequence when $\tilde{x}[n]$ is a symmetrical one. Otherwise, the number of independent values increases after decimation process.

In the following, we refer to the input sequence of the filter as $\tilde{x}[n]$, the output of the filter as $\tilde{g}[n]$, $\tilde{y}[n]$ as the output of the decimator and p denotes the arbitrary integer decimation factor.

A. Filtering and Symmetrical Sequence

Influence of the Filtering: First, we will consider about the influence of filtering on a symmetrically extended sequence. As the symmetric extension method requires the filters, which construct the filter banks, to be symmetric or antisymmetric [5], only linear phase FIR (LPFIR) filters are considered in the following.

The LPFIR filters can be classified into four cases [7] according to their impulse responses and they are shown in Fig. 4. When a symmetrical sequence $\tilde{x}[n]$ is filtered with a LPFIR filter, the output sequence $\tilde{g}[n]$ will also become a symmetrical sequence, and the period of $\tilde{x}[n]$ and $\tilde{g}[n]$ will be the same. Nevertheless, the type of symmetric sequence may be changed by the filtering. In Table II, the type of $\tilde{g}[n]$, the output sequence of the filter, is shown. In this table the relations marked by * (asterisk) have been previously reported. Therefore, Table II shows the more general relations including those pointed out in previous works.

Derivation of Table II: Using frequency domain representation the relations shown in Table II can be explained as follows. The frequency responses of the LPFIR filters can be expressed as [7]

$$H_1(e^{j\omega}) = e^{-j\omega\frac{N-1}{2}} H_1^*(e^{j\omega}) \quad N\text{: odd} \quad (5a)$$
$$H_2(e^{j\omega}) = e^{-j\omega\frac{N-1}{2}} H_2^*(e^{j\omega}) \quad N\text{: even} \quad (5b)$$
$$H_3(e^{j\omega}) = jH_1(e^{j\omega}) \quad (5c)$$
$$H_4(e^{j\omega}) = jH_2(e^{j\omega}) \quad (5d)$$

where $H_n^*(e^{j\omega})$ is purely real, and N denotes filter length. These

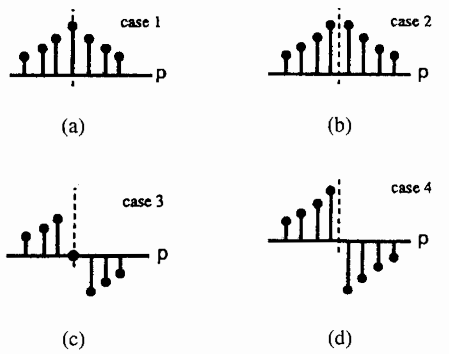

Fig. 4. Summary of types of linear phase FIR filters. (a) Even-order and symmetry. (b) Odd-order and symmetry. (c) Even-order and antisymmetry. (d) Odd-order and antisymmetry.

TABLE II
TYPE OF SYMMETRY OF FILTERED SEQUENCE

Sequence Type			Filter Type: Symmetry 1	Symmetry 2	Anti-symmetry 3	Anti-symmetry 4
	Symmetry	A	A	C *	D	F *
		B	B	B	E	E
		C	C *	A	F	D
	Anti-Symmetry	D	D	F	A	C
		E	E	E	B	B
		F	F	D	C	A

* Previously reported

TABLE III
THE VALUES OF TWO PARAMETERS B AND C FOR EACH TYPE OF LINEAR PHASE FIR FILTERS

Filter	B	C
1	Int.	$2n\pi$
2	Half Int.	$2n\pi$
3	Int.	$2n\pi + \pi/2$
4	Half Int.	$2n\pi + \pi/2$

n : integer
Int. : integer

equations can be unified in the next form. That is

$$H_n(e^{j\omega}) = e^{jC} e^{-j\omega B} H_n^*(e^{j\omega}). \tag{6}$$

where B and C are two parameters identifying the type of LPFIR filter. In Table III, we tabulate B and C for each type.

Using (4) and (6), we can derive the relations shown in Table II. Let us denote DFS of $\tilde{g}[n]$ as $\tilde{G}[k]$ and is expressed as

$$\begin{aligned}\tilde{G}[k] &= H(e^{j\omega})\tilde{X}[k] \\ &= e^{jC} e^{-j\omega B} H^*(e^{j\omega}) e^{jT} W_M^{kL} \tilde{X}^*[k] \\ &= e^{jC} e^{-j\omega_k B} H^*(e^{j\omega_k}) e^{jT} W_M^{kL} \tilde{X}^*[k] \\ &= e^{jC} e^{-j\omega_k B} e^{jT} W_M^{kL} \tilde{G}^*[k]\end{aligned} \tag{7}$$

where $\omega_k = 2\pi k/M$ and $\tilde{G}^*[k] = H^*(e^{j\omega_k})\tilde{X}^*[k]$. Therefore, we can obtain

$$\tilde{G}[k] = e^{j(C+T)} W_M^{k(B+L)} \tilde{G}^*[k]. \tag{8}$$

For example, let us consider the case that $\tilde{x}[n]$ is a case A sequence and $h[n]$ is a case 2 filter. Then $C + T = 0$, $B + L$ become an integer and M is even. It is known from Table I that the resulting sequence $\tilde{g}[n]$ becomes a case C sequence. We will show another example. When we consider that $\tilde{x}[n]$ is case F and $h[n]$ is case 4,

TABLE IV
TYPE OF SYMMETRY OF DECIMATED SEQUENCE

Sequence Type		P : even, M' : even	P : even, M' : odd	P : odd, M' : even	P : odd, M' : odd
A	Dec I	-	-	-	x
	Dec II	-	-	A	x
B	Dec I	x	x	x	B
	Dec II	x	x	x	-
C	Dec I	C	B	C	-
	Dec II	A	B	-	-
D	Dec I	-	-	-	x
	Dec II	-	-	D	x
E	Dec I	x	x	x	E
	Dec II	x	x	x	-
F	Dec I	F	E	F	-
	Dec II	D	E	-	-

Dec I : decimation I
Dec II : decimation II
x : conflict with condition (I) (see Sec.4)
- : conflict with condition (II)
P : decimation ratio
M' : the period of the decimated sequence

then $C + T = \pi$, $B + L$ becomes a half integer and M is even. Therefore, the resulting sequence becomes a case A sequence.

B. Down-Sampling and Symmetrical Sequences

When considering the effect of the decimation process on a symmetrical sequence we must pay attention to the next two points, namely

1) Symmetry may be lost after decimation.
2) Timing of decimation changes the type of symmetry.

Previously, we showed the results in Table IV. In the table, we tabulated the relations between P, the decimation ratio, M, the period of sequence and timing of decimation. We used the terms decimation I and II to show the different decimation timing in the table and they are discussed in detail later.

Derivation of Table IV: Let us formulate the relations between $\tilde{g}[n]$ and $\tilde{y}[m]$. We assume that the original sequence $x[n]$ starts from $t = 0$. Let M denotes the period of $\tilde{x}[n]$ and P denotes the integer decimation factor, then they have a relation $M = M'P$, where M' is the period of a decimated sequence $\tilde{y}[m]$.

We define the sequence $\tilde{g}'[n]$ as

$$\tilde{g}'[n] = \begin{cases} \tilde{g}[n-s] & n = 0, P, 2P, \cdots \\ 0 & \text{otherwise.} \end{cases} \tag{9}$$

where s represents a shift value which determines the timing of decimation. Then we get

$$\tilde{y}[m] = \tilde{g}'[Pm] = \tilde{g}[Pm - s]. \tag{10}$$

Using these definitions, DFS of $\tilde{y}[m]$ can be written as

$$\begin{aligned}\tilde{Y}[k] &= \sum_{m=0}^{M'-1} W_{M'}^{km} \\ &= \frac{1}{M} \sum_{q=0}^{P-1} W_M^{s(k-M\ell/P)} \tilde{G}\left[k - \frac{M\ell}{P}\right]\end{aligned} \tag{11}$$

where $\tilde{G}[k]$ is DFS of $\tilde{g}[n]$. By substituting (3a) into (11), we can obtain the formulation for $\tilde{y}[k]$ in case of $\tilde{g}[n]$ being a case A sequence or a case B as

$$\tilde{Y}[k] = \frac{1}{P} W_{M'}^{\frac{(M'+b)k}{2}} \sum_{m=0}^{P-1} \exp\left[j\pi m(M'+b)\right] \tilde{G}^*[k - M'm] \tag{12}$$

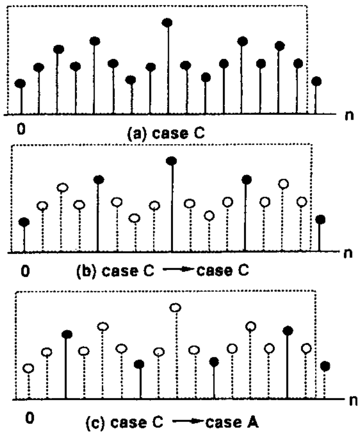

Fig. 5. The timing of decimation. (a) Orignal sequences. (b) A sequence generated by Decimation I. (c) A sequence generated by Decimation II.

where $b = (2s-1)/P$. For a case C sequence, we obtain from (3c)

$$\tilde{Y}[k] = \frac{1}{P} W_{M'}^{\frac{(M'+b')k}{2}} \sum_{m=0}^{P-1} \exp\left[j\pi m(M'+b')\right]\tilde{G}^*[k-M'm] \tag{13}$$

where $b' = 2s/P$.

Timing of Decimation: Here we consider (12) in the form of (4). For that, the exponential term in the summation must be real. This requires each of two variables

$$M' = \frac{M}{P} \tag{14}$$

and

$$b = \frac{2s-1}{P} \qquad s = 0, 1, \cdots, M-1 \tag{15}$$

to be an integer. On the other hand, from (13)

$$M' = \frac{M}{P} \tag{16}$$

and

$$b' = \frac{2s}{P} \qquad s = 0, 1, \cdots, M-1 \tag{17}$$

must be integers for writing (13) in the form of (4).

From (14) and (16), we can see that M must be an integer multiple of P. From (15) and (17), we can see the values that s can take are limited, or it depends on P. This means that when P is given the timing of decimation it generates a new symmetrical sequence which is restricted.

In general, we can distinguish two different decimation timings that may create a symmetrical sequence. They can be identified by whether they leave the value corresponding to the center of symmetry or not. Let us define Decimation I as decimation which leaves the value, and Decimation II as that which does not. We show them in Fig. 5 where $\tilde{g}[n]$ is case C.

IV. Discussion

So far, we have described the influence of analysis process on a symmetrically extended sequence. We tabulate types of filtered sequences In Table II and that of decimated sequences in Table IV.

We can remove the restriction of the previous work by utilizing these two tables. When the number of divisions P is given then from Table IV we can know what type of sequence can be used as $\tilde{g}[n]$, the input to a decimator. To obtain the desired type of sequences after filtering process, we must select the type of $\tilde{x}[n]$ according to Table II depending on the filters consisting the filter banks. Thus the symmetric extension method can be applied to the system of an arbitrary number of divisions.

V. Conclusion

In this paper, we developed a technique for high quality subband image coding at low bit rates proposed by Smith *et al.* We proposed six methods of generating symmetrical periodic sequences and investigated their behavior in the analysis/synthesis system. Especially, the influence of decimation process on a symmetric sequence was considered in detail. For that, we used the frequency domain representations to formulate the relations, so that the theory can be clearly developed.

We have derived the conditions for using the symmetric extension method in the general structure of an analysis/synthesis system.

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