A NEW LIFTING STRUCTURE OF NON SEPARABLE 2D DWT WITH COMPATIBILITY TO JPEG 2000

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ABSTRACT

In this report, we reduce the number of lifting steps of the two dimensional (2D) discrete wavelet transform (DWT) in JPEG 2000 by introducing a factorization of the separable 2D transfer function into some non separable 2D (NS-2D) functions. The order of the 1D lifting steps in vertical and in horizontal are properly replaced so that all of them are synthesized into NS-2D functions. As a result, the number of the lifting steps of the 2D 9/7 DWT is reduced to 75 [%] maintaining compatibility to the JPEG 2000. Since the number of the lifting steps is essentially proportional to a delay of output signals, it contributes to construct a low latency DWT under the assumption that direct accessing to a 2D intermediate memory is permitted.

***Index Terms*—** image, wavelet, coding, JPEG 2000

1. INTRODUCTION

Over the past few decades, a considerable number of studies have been conducted on the discrete wavelet transform (DWT). Ever since the digital cinema adopted the JPEG 2000 international standard, there has been a renewal of interest in the lifting based DWT, especially on attaining high throughput and low latency of the DWT processing for high definition video signal coding [1,2].

Some of the most compelling studies have focused on hardware-efficient implementation of the lifting DWT. Intermediate memory utilization has been well studied by introducing the line based implementation [3]. A lifting factorization has been proposed to reduce auxiliary buffers to increase throughput for boundary processing in the block based DWT [4]. The parallel and pipeline techniques in the folded architecture have been studied to increase hardware utilization and to reduce the critical path latency [5,6]. However, in the lifting DWT, a delay of output signals is essentially determined by the number of the lifting steps. Therefore, most of these hardware implementation techniques are limited by this delay (lifting step latency).

Unlike those prior works, we discuss on how to reduce the number of the lifting steps of the two dimensional (2D) DWT, under the assumption that direct accessing to a 2D intermediate memory is permitted. In this report, we introduce a factorization of the separable 2D transfer function of the DWT into non separable 2D (NS-2D) transfer functions.

So far, quite a few NS-2D factorization techniques have been proposed [7-9]. The residual correlation of the Haar transform has been utilized by the NS-2D lifting scheme [7]. The Walsh Hadamard transform has been composed of a NS-2D lossless transform and applied to construct a lossless discrete cosine transform (DCT) [8]. A NS-2D adaptive lifting scheme with a morphological operation has been reported in [9]. However, these transforms are not compatible to the JPEG 2000.

In this report, we reduce the number of the lifting steps of the 2D 9/7 DWT in JPEG 2000 by expanding our previous work for the 5/3 DWT reported in [10]. Since the 9/7 DWT has doubled lifting steps comparing to the 5/3 DWT, its importance cannot be overemphasized for implementation of essentially high throughput processing.

We properly replace the order of the 1D lifting steps in vertical and in horizontal so that all of them are synthesized into NS-2D functions. Although the proposed DWT requires direct accessing to a 2D intermediate memory, it is perfectly compatible to the JPEG 2000 and also it has lowered lifting step latency.

2. TWO DIMENSIONAL 5/3 DWT

2.1. Cascade of 1D Processing

Fig.1(a) illustrates the forward transform of the existing 2D 5/3 DWT in one level. It splits the input signal X into four subset signals X_{11}, X_{21}, X_{12} and X_{22} by the down sampler with $\mathbf{D}=diag[2\ 2]$.

In the lifting part, the signals X_{12} and X_{22} are predicted by X_{11} and X_{21} respectively with a vertical filter $H_1(z_1)$ where z_1 denotes the one sample delay as illustrated in Fig.2(a). These predictions can be implemented simultaneously in parallel. We refer to this stage as the 1st step.

In the 2nd step, X_{11} and X_{21} are updated by X_{12} and X_{22} inversely with a filter $H_2(z_1)$. Note that the 2nd step can't be performed before the 1st step is completed. It causes a delay and we refer to this delay as the lifting path latency.

After the prediction and updating in vertical, the same processing is applied in horizontal. As a result, the four band signals *LL*, *LH*, *HL* and *HH* are produced by

$$\mathbf{Y} = (\mathbf{L}_{\mathrm{H}_2^*,\mathrm{H}_1^*}\mathbf{P}_{23})(\mathbf{L}_{\mathrm{H}_2,\mathrm{H}_1}\mathbf{P}_{23})\mathbf{X} \qquad (1)$$

where

$$\mathbf{X} = \begin{bmatrix} X_{11} \\ X_{21} \\ X_{12} \\ X_{22} \end{bmatrix}, \ \mathbf{Y} = \begin{bmatrix} LL \\ LH \\ HL \\ HH \end{bmatrix}, \ \mathbf{P}_{23} = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix},$$

$$\mathbf{L}_{p,q} = \begin{bmatrix} \begin{bmatrix} 1 & p \\ 0 & 1 \end{bmatrix}\begin{bmatrix} 1 & 0 \\ q & 1 \end{bmatrix} & \begin{bmatrix} 0 & 0 \\ 0 & 0 \end{bmatrix} \\ \begin{bmatrix} 0 & 0 \\ 0 & 0 \end{bmatrix} & \begin{bmatrix} 1 & p \\ 0 & 1 \end{bmatrix}\begin{bmatrix} 1 & 0 \\ q & 1 \end{bmatrix} \end{bmatrix}$$

for $p,q \in \{\mathrm{H}_1, \mathrm{H}_2, \mathrm{H}_1^*, \mathrm{H}_2^*\}$ and the filters are defined by

$$[\mathrm{H}_n \quad \mathrm{H}_n^*] = [H_n(z_1) \quad H_n(z_2)\ ]\ , n \in \{1,2\}\ ,$$
$$[H_1(z) \quad H_2(z)\ ] = [h_1 \quad h_2][1+z^{-1} \quad 1+z]^T\ .$$

As summarized in Fig.2(a), the existing DWT in Fig.1(a) requires four lifting steps per one level of the octave decomposition. Our purpose in this report is to reduce the number of the lifting steps so that the lifting path latency of the DWT is lowered, under the assumption that accessing to a 2D intermediate memory is permitted.

2.2. 2D Direct Processing

Fig.1(b) illustrates the 2D 5/3 DWT with reduced lifting steps. Note that it requires only three lifting steps and its output signals are exactly the same as those of the existing 5/3 DWT in Fig.1(a). It is based on the following theorem.

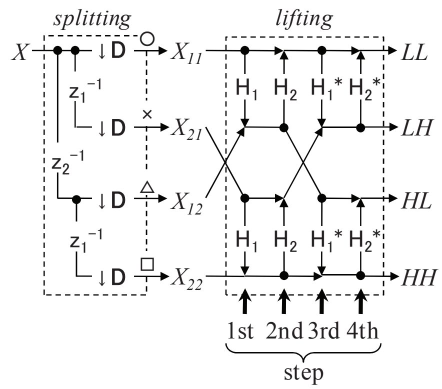
Theorem 1:

$$\mathbf{L}_{d,c}\mathbf{P}_{23}\mathbf{L}_{b,a}\mathbf{P}_{23} = \mathbf{N}_{d,c,b,a} \qquad (2)$$

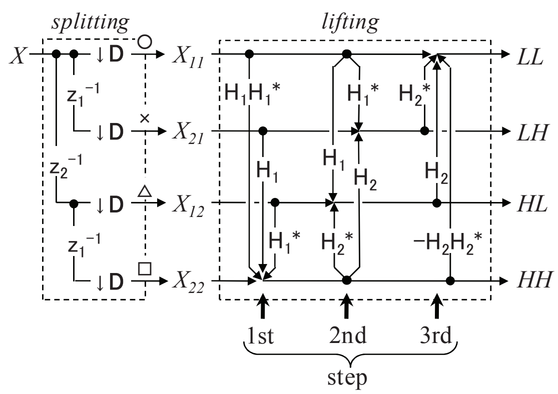
where

$$\mathbf{N}_{d,c,b,a} = \begin{bmatrix} 1 & d & b & -bd \\ c & 1 & 0 & b \\ a & 0 & 1 & d \\ ac & a & c & 1 \end{bmatrix}.$$

Its directions of the prediction and updating are illustrated in Fig.2(b). It requires direct accessing to a 2D intermediate memory. We have previously proposed this structure in [10]. In this report, we expand this idea to the 2D 9/7 DWT in section **3.2**.

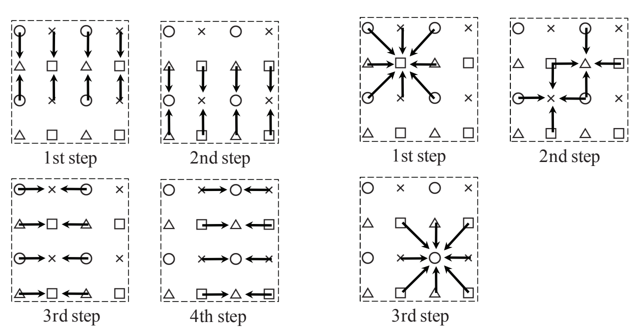


(a) Cascade of 1D



(b) 2D direct

Fig.1 Block diagram of the 2D 5/3 DWT in one level.



(a) Cascade of 1D (b) 2D direct

Fig.2 Directions of the prediction and updating.

3. TWO DIMENSIONAL 9/7 DWT

3.1. Cascade of 1D Processing

Fig.3(a) illustrates the existing 2D 9/7 DWT in one level (four band decomposition). Comparing to the 5/3 DWT, it has two more lifting steps in each of row and column. Signals are scaled with k and k^{-1} to keep the DC gain to be one. Its output signals are calculated by

$$\mathbf{Y} = (\mathbf{J}_k \mathbf{L}_{\mathrm{H}_4^*,\mathrm{H}_3^*} \mathbf{L}_{\mathrm{H}_2^*,\mathrm{H}_1^*} \mathbf{P}_{23})(\mathbf{J}_k \mathbf{L}_{\mathrm{H}_4,\mathrm{H}_3} \mathbf{L}_{\mathrm{H}_2,\mathrm{H}_1} \mathbf{P}_{23})\mathbf{X} \quad (3)$$

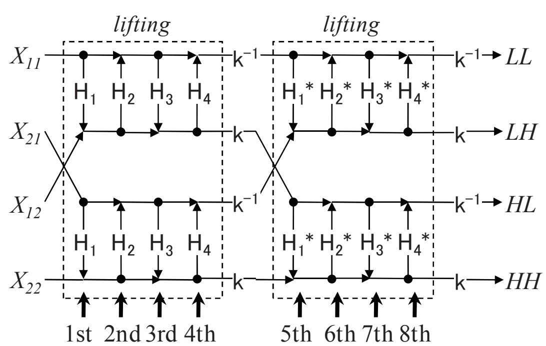
where

$$\mathbf{J}_k = diag[k^{-1} \quad k \quad k^{-1} \quad k \ ],$$

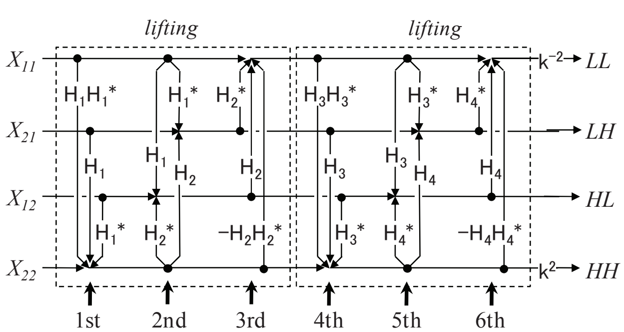
$$[\mathrm{H}_n \quad \mathrm{H}_n^*] = [H_n(z_1) \quad H_n(z_2) \ ] \ ,$$

$$[H_{2m-1}(z) \quad H_{2m}(z)] = [h_{2m-1} \quad h_{2m}][1+z^{-1} \quad 1+z]^T \ ,$$

for $n \in \{1,2,3,4\}$ and $m \in \{1,2\}$. Note that this structure has eight lifting steps per level.



(a) Cascade of 1D



(b) Proposed 2D direct

Fig.3 Block diagram of the 2D 9/7 DWT in one level.

3.2. Proposed 2D Direct Processing

Fig.3(b) illustrates the proposed 2D 9/7 DWT in one level. Comparing to the existing DWT in Fig.3(a), the number of the lifting steps is reduced to six (minus two steps), and the scaling procedures are synthesized to k^2 and k^{-2}. Its output signals are calculated by

$$\mathbf{J}_k^* \mathbf{N}_{\mathrm{H}_4^*,\mathrm{H}_3^*,\mathrm{H}_4,\mathrm{H}_3} \mathbf{N}_{\mathrm{H}_2^*,\mathrm{H}_1^*,\mathrm{H}_2,\mathrm{H}_1} \quad (4)$$

where

$$\mathbf{J}_k^* = diag[k^{-2} \quad 1 \quad 1 \quad k^2 \ ].$$

This form is an expansion of the idea described in **2.2** to the 2D 9/7 DWT. Derivation process is explained in the next section.

3.3. Derivation of the Proposed Processing

Firstly, we unify the four scaling pairs [k^{-1} k] in Fig.3(a) to only one pair [k^{-2} k^{2}] as illustrated in Fig.4(a). It is described as

$$\begin{aligned}&(\mathbf{J}_k \mathbf{L}_{\mathrm{H}_4^*,\mathrm{H}_3^*} \mathbf{L}_{\mathrm{H}_2^*,\mathrm{H}_1^*} \mathbf{P}_{23})(\mathbf{J}_k \mathbf{L}_{\mathrm{H}_4,\mathrm{H}_3} \mathbf{L}_{\mathrm{H}_2,\mathrm{H}_1} \mathbf{P}_{23}) \\ &= \mathbf{J}_k^* \mathbf{L}_{\mathrm{H}_4^*,\mathrm{H}_3^*} \mathbf{L}_{\mathrm{H}_2^*,\mathrm{H}_1^*} \mathbf{P}_{23} \mathbf{L}_{\mathrm{H}_4,\mathrm{H}_3} \mathbf{L}_{\mathrm{H}_2,\mathrm{H}_1} \mathbf{P}_{23}\end{aligned} . \quad (5)$$

Secondly, applying the following theorem,

Theorem 2:

$$\mathbf{L}_{\mathrm{H}_s^*,\mathrm{H}_r^*} \mathbf{P}_{23} \mathbf{L}_{\mathrm{H}_q,\mathrm{H}_p} \mathbf{P}_{23} = \mathbf{P}_{23} \mathbf{L}_{\mathrm{H}_q,\mathrm{H}_p} \mathbf{P}_{23} \mathbf{L}_{\mathrm{H}_s^*,\mathrm{H}_r^*} \quad (6)$$

to the lifting steps in a broken line in Fig.4(b), we replace the order of the processing in row and in column as illustrated in Fig.4(c). It is described as

$$\begin{aligned}&\mathbf{L}_{\mathrm{H}_4^*,\mathrm{H}_3^*}(\mathbf{L}_{\mathrm{H}_2^*,\mathrm{H}_1^*} \mathbf{P}_{23} \mathbf{L}_{\mathrm{H}_4,\mathrm{H}_3} \mathbf{P}_{23}) \mathbf{P}_{23} \mathbf{L}_{\mathrm{H}_2,\mathrm{H}_1} \mathbf{P}_{23} \\ &= \mathbf{L}_{\mathrm{H}_4^*,\mathrm{H}_3^*}(\mathbf{P}_{23} \mathbf{L}_{\mathrm{H}_4,\mathrm{H}_3} \mathbf{P}_{23} \mathbf{L}_{\mathrm{H}_2^*,\mathrm{H}_1^*}) \mathbf{P}_{23} \mathbf{L}_{\mathrm{H}_2,\mathrm{H}_1} \mathbf{P}_{23}\end{aligned} . \quad (7)$$

Finally, applying the theorem 1 to each of the groups in the broken line in Fig.4(d), we have the proposed structure in Fig.3(b). It is described as

$$\begin{aligned}&(\mathbf{L}_{\mathrm{H}_4^*,\mathrm{H}_3^*} \mathbf{P}_{23} \mathbf{L}_{\mathrm{H}_4,\mathrm{H}_3} \mathbf{P}_{23})(\mathbf{L}_{\mathrm{H}_2^*,\mathrm{H}_1^*} \mathbf{P}_{23} \mathbf{L}_{\mathrm{H}_2,\mathrm{H}_1} \mathbf{P}_{23}) \\ &= \mathbf{N}_{\mathrm{H}_4^*,\mathrm{H}_3^*,\mathrm{H}_4,\mathrm{H}_3} \mathbf{N}_{\mathrm{H}_2^*,\mathrm{H}_1^*,\mathrm{H}_2,\mathrm{H}_1}\end{aligned} . \quad (8)$$

As a result, the proposed 2D 9/7 DWT in Fig.3(b) is derived from the existing DWT in Fig.3(a). Is has exactly the same transfer function of the JPEG 2000. Note that the number of the lifting steps is reduced from eight to six (75 %). This reduction rate is the same for the multi level dyadic decomposition when the proposed DWT is cascaded. It contributes to decrease the lifting path latency of the 2D 9/7 DWT.

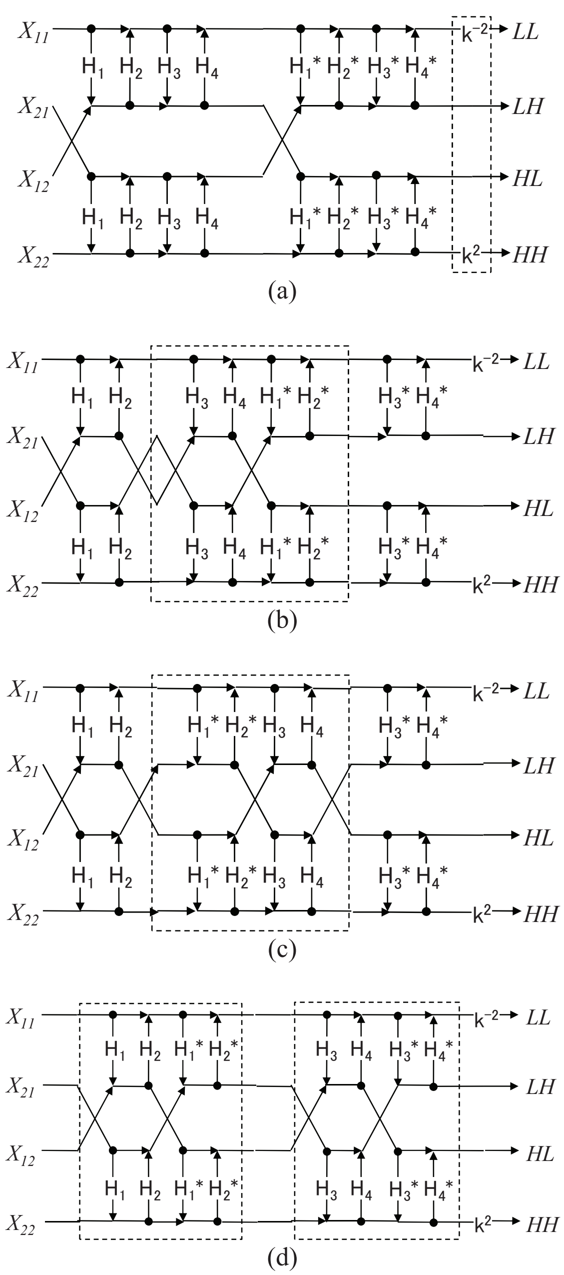


Fig.4 Derivation of the proposed 2D 9/7 DWT.

4. CONCLUSIONS

In this report, we factorized the separable 2D transfer function of the 2D 9/7 DWT in JPEG 2000 into some non separable 2D functions. We properly replaced the order of the 1D lifting steps in vertical and in horizontal and synthesized all of them into non separable 2D functions with reduced lifting steps. As a result, the number of the lifting steps which we referred to the lifting step latency of the 2D DWT is reduced to 75 [%]. It also maintains the perfect compatibility to the JPEG 2000.

Although the proposed DWT requires direct accessing to a 2D intermediate memory, its lowered lifting step latency essentially contributes to construct a high throughput 2D 9/7 DWT for an image signal processing.

5. REFERENCES

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