

A LOSSLESS CONDITION OF LIFTING DWT FOR SPECIFIC DC VALUES

Masahiro IWAHASHI and Hitoshi KIYA

Nagaoka University of Technology
Niigata, 980-2188, Japan.

Tokyo Metropolitan University
Tokyo, 191-0065, Japan

ABSTRACT

This report theoretically derives a condition on the word length of coefficients and signals such that the irreversible 9-7 discrete wavelet transform (DWT) in JPEG 2000 becomes lossless for a constant valued (DC) input signal. Under the condition, the output signal contains no error in spite of rounding of coefficients and signals inside the DWT circuit. We have previously analyzed the condition for any value of DC signals. However, it was too strict and the word length under the condition was redundant for a practical case. In this report, we derive a new condition for a specific DC value and show that the minimum word length is halved under the new condition.

Index Terms— image, wavelet, coding, lossless

1. INTRODUCTION

Since the JPEG 2000 based on the discrete wavelet transform (DWT) has been adopted as an international standard for digital cinema, high speed and low power implementation of the DWT has been becoming an issue of great importance [1-4]. In design of the DWT, its coefficients and signals are assumed to be real numbers. However, in implementation, they are rounded to rational numbers so that they are expressed with finite word length representation. Therefore it is inevitable to have the rounding errors inside the DWT circuit [5-7].

In this report, we theoretically analyze the condition on the word length of coefficient values and signal values such that the irreversible 9-7 DWT becomes lossless for a constant valued input signal (DC signal). Under the condition, the output signal of the DWT contains no error (DC lossless). This property is considered to be effective for the white balancing of an image processing system [8,9].

In conventional analysis, the error due to shortening the word length of signals (signal error) was described as additive to the signal [5]. It was treated as an independent and uniformly distributed white signal. On the other hand, the error of coefficients (coefficient error) was described as multiplicative to the signal and evaluated with the sensitivity [6]. We have analyzed the errors of a new class of the lifting DWT [7]. However, all of them treated the

signal error and the coefficient error independently and their mutual effect has not been well studied.

In this report, introducing a new model which shifts the coefficient error to the signal error, we take their mutual effect into account. We have previously analyzed the condition for any value of DC signals [9]. However, it was too strict and the word length under the condition was redundant for a practical case. In this report, we derive a new condition for a specific DC value. We also show that the minimum word length is halved under the new condition.

2. DC LOSSLESS DWT AND ITS WORD LENGTH

2.1. Irreversible 9-7 DWT

Fig.1 illustrates the irreversible 9-7 DWT in the JPEG 2000 [1]. The input signal $x(n)$, $n=\{1,2,\dots,N\}$ is transformed to the band signals $y_1(m)$ and $y_2(m)$, $m=\{1,2,\dots,N/2\}$. These are backward transformed to reconstruct the signal $w(n)$. In the figure, z^{-1} and $\downarrow 2$ indicate the delay and the down sampler respectively. The coefficients c_i , $i \in \{1,2,\dots,6\}$ of multipliers are designed as real numbers. In implementation, their word lengths are shortened. The fraction part of a signal value is also truncated to F_S , F_B or F_X [bit] by the rounding operation illustrated as a circle in the figure.

2.2. Word Length and Rounding Error

In this report, we use the rounding operation defined as

$$\begin{cases} R_0[s] = \lfloor s' \rfloor = s' - (s' \bmod 1) \in \mathbf{Z}, & s' = s + 2^{-1} \\ R_{F_S}[s] = R_0[s2^{F_S}]2^{-F_S}, & 0 \leq F_S \in \mathbf{Z} \end{cases} \quad (1)$$

as an example. The operation $R_{F_S}[\]$ shortens the fraction part of the word length of a signal value s into F_S [bit]. It also generates the error:

$$\Delta_{F_S}[s] = s - R_{F_S}[s]. \quad (2)$$

Denoting the integer part as I_S [bit], the word length W_S [bit] of a signal s is defined as

$$W_S = I_S + F_S + 1 \text{ [bit]} \quad (3)$$

including 1 [bit] for the sign part. Similarly, the word length W_C [bit] of a coefficient c is defined as

$$W_C = I_C + F_C + 1 \text{ [bit]}. \quad (4)$$

Especially, in this report, we utilize the property [9]:

$$\begin{cases} R_{F_S}[s]2^{F_S} = p \in \mathbf{Z} \text{ and } |\Delta_{F_S}[s]| \leq 2^{-1-F_S} \text{ for } \forall s \\ R_{F_S}[s]2^{F_S} \leq p \Leftrightarrow |s| < (p+2^{-1})2^{-F_S} \text{ for } \forall s \\ s2^{F_S} \in \mathbf{Z} \Rightarrow R_{F_S}[s+t] = s + R_{F_S}[t] \end{cases} \quad (5)$$

to analyze the DC lossless condition on the word length.

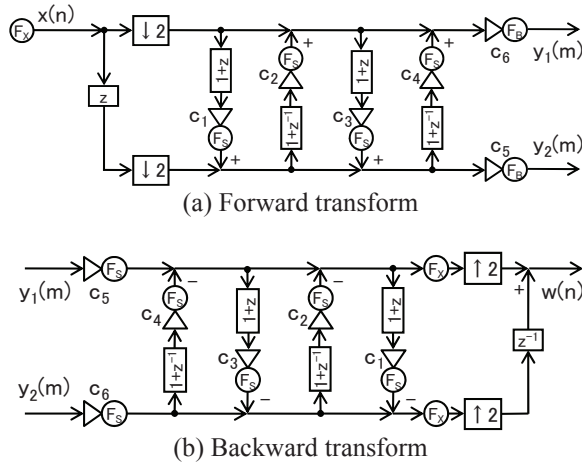


Fig.1 Irreversible 9-7 DWT.

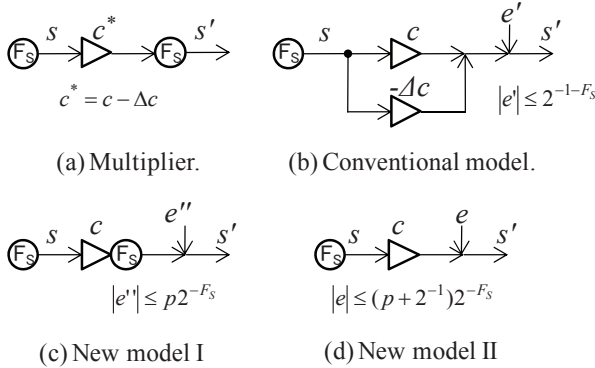


Fig. 2 A multiplier in the DWT and its models for analysis.

2.3. DC Lossless DWT

In an image processing system, an input signal is processed through a camera, a pair of an encoder and a decoder, and a display. When the camera and the display are adjusted, a white balancing signal, which is equivalent to a constant

valued signal or a DC signal, is commonly used [8]. In this case, it is desirable that the encoder and the decoder do not generate any loss in its output signals.

In this report, we define the loss as the difference between the output signal of the DWT with infinite word length and that of the DWT with shortened word length. For DC input, when the output of the backward transform (reconstructed signal) $w(n)$ becomes lossless, and also the output of the forward transform (band signal) $[y_1(m) y_2(m)]$ become lossless, we refer to it as the DC lossless.

3. ANALYSIS ON WORD LENGTH CONDITION

3.1. Shifted Error Model for Analysis

Fig.2 (a) illustrates a multiplier in the DWT circuit. A coefficient value c designed as a real number is rounded to a rational number c^* in implementation. The fraction part of both of the input signal s and the output signal s' is rounded to F_S [bit]. These are denoted as

$$\begin{cases} s' = R_{F_S}[c^*s], s2^{F_S} \in \mathbf{Z} \\ c^* = c - \Delta c, c^* = R_{F_C}[c], \Delta c = \Delta_{F_C}[c] \end{cases} \quad (6)$$

A conventional model for error analysis is illustrated in Fig.2 (b). It describes the coefficient error $-\Delta c \cdot s$ as multiplicative to the signal s [6], and the signal error e' as additive [5]. These are treated independently as

$$s' = cs - \Delta c \cdot s + e', s2^{F_S} \in \mathbf{Z}, |e'| \leq 2^{-1-F_S}. \quad (7)$$

On the contrary, as illustrated in Fig.2 (c), we describe the coefficient error e'' as

$$\begin{cases} s' = R_{F_S}[cs] + e'', s2^{F_S} \in \mathbf{Z} \\ e'' = R_{F_S}[\Delta_{F_S}[cs] - \Delta_{F_C}[c]s] \end{cases} \quad (8)$$

We utilized the fact that the coefficient error e'' is observed as additive to the signal when both of the coefficients and signals are rounded. It should be noticed that e'' in Eq.(8) is not an approximation but a strictly described value. This model can be denoted as

$$\begin{cases} s' = cs + e, s2^{F_S} \in \mathbf{Z} \\ e = e' + e'' \\ e' = -\Delta_{F_S}[cs] \\ e'' = R_{F_S}[-\Delta_{F_C}[c]s + \Delta_{F_S}[cs]] \end{cases} \quad (9)$$

as illustrated in Fig.2 (d). Finally, the coefficient error e'' is shifted to the signal error e (shifted error) and it becomes possible to derive the word length condition considering mutual effect of the coefficient error and the signal error.

3.2. DC Equivalent Circuit

When the input is restricted to DC signals, $x(n)$ can be described as a scalar x independent of n . The delay z^{-1} can be treated as 1 and $(1+z^{-1})$ can be replaced by 2. Therefore, instead of the circuits in Fig.1, we use the equivalent circuits for DC signals in Fig.3 to derive the condition.

In Fig.3 (a), a scalar x with F_X [bit] fraction part is multiplied by the rational numbers c_i and rounded to F_S [bit]. Finally, the signals are rounded to F_B [bit] at its output to produce the band signals $[y_1 y_2]$. The shifted errors inside the circuit are described as

$$\begin{aligned} e_i &= e_i' + e_i'' , \quad i \in \{1,2,3,4,5,6\} \\ \begin{cases} e_i' = -\Delta_{F_S} [c_i s_i] , \\ e_i'' = R_{F_S} [\Delta_{F_S} [c_i s_i] - \Delta_{F_C} [c_i] s_i] , \end{cases} \end{aligned} \quad (10)$$

where

$$\begin{aligned} \begin{bmatrix} s_1 & s_3 & s_5 \\ s_2 & s_4 & s_6 \end{bmatrix} &= \begin{bmatrix} 2x & 2(x+s'_2) & s_4 \\ 2(x+s'_1) & 2(s_2+s'_3) & s_3+s'_4 \end{bmatrix} , \\ s'_i &= c_i s_i + e_i = R_{F_S} [c_i s_i] . \end{aligned}$$

For the backward transform in Fig.3 (b), signals and errors are similarly described as

$$\begin{aligned} f_i &= f_i' + f_i'' , \quad i \in \{1,2,3,4,5,6\} \\ \begin{cases} f_i' = -\Delta_{F_S} [c_i t_i] \\ f_i'' = R_{F_S} [\Delta_{F_S} [c_i t_i] - \Delta_{F_C} [c_i] t_i] \end{cases} \end{aligned} \quad (11)$$

where

$$\begin{aligned} \begin{bmatrix} t_1 & t_3 & t_5 \\ t_2 & t_4 & t_6 \end{bmatrix} &= \begin{bmatrix} 2(t_3-t'_2) & 2(t'_5-t'_4) & y_1 \\ 2(t_4-t'_3) & 2t'_6 & y_2 \end{bmatrix} , \\ t'_i &= c_i t_i + f_i = R_{F_S} [c_i t_i] . \end{aligned}$$

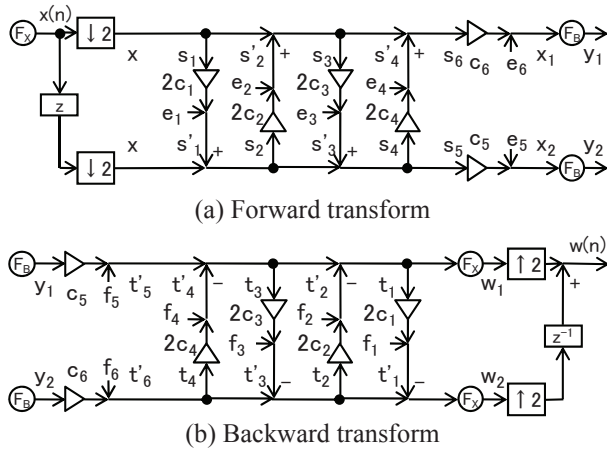


Fig. 3 Equivalent circuits of the DWT for DC input signals.

3.3. Condition on Word Length for DC Lossless DWT

In Fig.3(a), the shifted errors in Eq.(10) are accumulated in the circuit. When the accumulated errors are nullified by the rounding at output of the transform, the DWT becomes DC lossless. In the figure, $\mathbf{Y}_{12}=[y_1 y_2]^T$ is described as

$$\begin{aligned} \mathbf{Y}_{12} &= R_{F_B} [\mathbf{I}_U e_6 + \mathbf{I}_L e_5 + \mathbf{K}(\mathbf{I}_U e_4 + \mathbf{H}_4(\mathbf{I}_L e_3 \\ &+ \mathbf{H}_3(\mathbf{I}_U e_2 + \mathbf{H}_2(\mathbf{I}_L e_1 + \mathbf{H}_1 \mathbf{I}_{UL} x)))] \end{aligned} \quad (12)$$

where

$$\begin{aligned} \mathbf{I}_U &= [1 \ 0]^T, \quad \mathbf{I}_L = [0 \ 1]^T, \quad \mathbf{I}_{UL} = \mathbf{I}_U + \mathbf{I}_L \\ \mathbf{H}_{i \in \{1,3\}} &= \begin{bmatrix} 1 & 0 \\ 2c_i & 1 \end{bmatrix}, \quad \mathbf{H}_{j \in \{2,4\}} = \begin{bmatrix} 1 & 2c_j \\ 0 & 1 \end{bmatrix}, \quad \mathbf{K} = \begin{bmatrix} c_6 & 0 \\ 0 & c_5 \end{bmatrix} . \end{aligned}$$

It is described with the shifted errors \mathbf{E}_1 and \mathbf{E}_2 as

$$\mathbf{Y}_{12} = R_{F_B} [(\mathbf{H}_{e1} \mathbf{E}_1 + \mathbf{H}_{e2} \mathbf{E}_2) + \mathbf{K} \mathbf{H}_{4321} \mathbf{x}] \quad (13)$$

where

$$\begin{cases} \mathbf{H}_{e1} = [\mathbf{I}_U & \mathbf{K} \mathbf{I}_U & \mathbf{K} \mathbf{H}_{43} \mathbf{I}_U] \\ \mathbf{H}_{e2} = [\mathbf{I}_L & \mathbf{K} \mathbf{H}_4 \mathbf{I}_L & \mathbf{K} \mathbf{H}_{432} \mathbf{I}_L] \\ \mathbf{H}_{43\dots} = \mathbf{H}_4 \mathbf{H}_3 \dots \end{cases}, \quad \begin{cases} \mathbf{E}_1 = [e_6 & e_4 & e_2]^T \\ \mathbf{E}_2 = [e_5 & e_3 & e_1]^T \end{cases} .$$

In case of the DC lossless, it becomes

$$\hat{\mathbf{Y}}_{12} = \mathbf{K} \mathbf{H}_{4321} \mathbf{x} = \mathbf{I}_U \mathbf{x} . \quad (14)$$

Therefore, from Eq.(5), the loss in the band signals becomes

$$\mathbf{E}_{y_{12}} = \mathbf{Y}_{12} - \hat{\mathbf{Y}}_{12} = R_{F_B} [(\mathbf{H}_{e1} \mathbf{E}_1 + \mathbf{H}_{e2} \mathbf{E}_2)] \quad (15)$$

and the DC lossless condition on the forward transform for an input DC value x is derived as

$$|\mathbf{H}_{e1} \mathbf{E}_1 + \mathbf{H}_{e2} \mathbf{E}_2| < 2^{-1-F_B} . \quad (16)$$

Similarly, the DC lossless condition on the backward transform in Fig.3 (b) is derived as

$$|\mathbf{H}_{e3} \mathbf{E}_3 + \mathbf{H}_{e4} \mathbf{E}_4| < 2^{-1-F_X} . \quad (17)$$

where

$$\begin{cases} \mathbf{H}_{e3} = -[\mathbf{H}_1^{-1} \mathbf{I}_U & \mathbf{H}_{123}^{-1} \mathbf{I}_U & \mathbf{H}_{1234}^{-1} \mathbf{I}_U] \\ \mathbf{H}_{e4} = -[\mathbf{I}_L & \mathbf{H}_{12}^{-1} \mathbf{I}_L & \mathbf{H}_{1234}^{-1} \mathbf{I}_L] \\ \mathbf{H}_{12\dots i}^{-1} = \mathbf{H}_1^{-1} \mathbf{H}_2^{-1} \dots \mathbf{H}_i^{-1} \end{cases}, \quad \begin{cases} \mathbf{E}_3 = [f_2 & f_4 & f_5]^T \\ \mathbf{E}_4 = [f_1 & f_3 & f_6]^T \end{cases} .$$

Finally, the DC lossless condition is derived as Eq.(16) and (17) for an arbitrary input DC input value x .

4. SIMULATION RESULTS

We investigate the optimum word length of the coefficients and signals in the DWT which satisfy the DC lossless condition derived in 3.3.

The bold lines in Fig.4 indicate the theoretical lower bound of the word length for any kind of input DC value x . It is derived by applying the property in Eq.(5) to the condition Eq.(16) and Eq.(17) [9]. It is described as

$$-\log_2(2^{-\Delta W_c} + 2^{-\Delta W_s}) > G_E \quad [\text{bit}] \quad (18)$$

where

$$[\Delta W_C \quad \Delta W_S] = [F_C - I_S \quad F_S], G_E = 2.66.$$

Focusing on the case where the cost function $J=(F_C+F_S)$ is minimized as an example, the theoretical curve in Fig.4 (a) shows that the optimum word length must satisfy $F_S > 3.66$ and $F_C > 11.66$ for the 8 bit system with $(I_x, F_x)=(8, 0)$, namely $(F_S, F_C)=(4, 12)$ and $J=16$ [bit] are the theoretically derived optimum word length for any possible value of x .

The circles in Fig.4 (a) illustrates a pair of (F_S, F_C) at which the DWT becomes DC lossless for an integer x in the interval $[0, 255]$ (128 is initially subtracted). In this case, the optimum word length is $(F_S, F_C)=(2, 12)$. Note that the word length of signals F_S is shortened from 4 to 2 [bit].

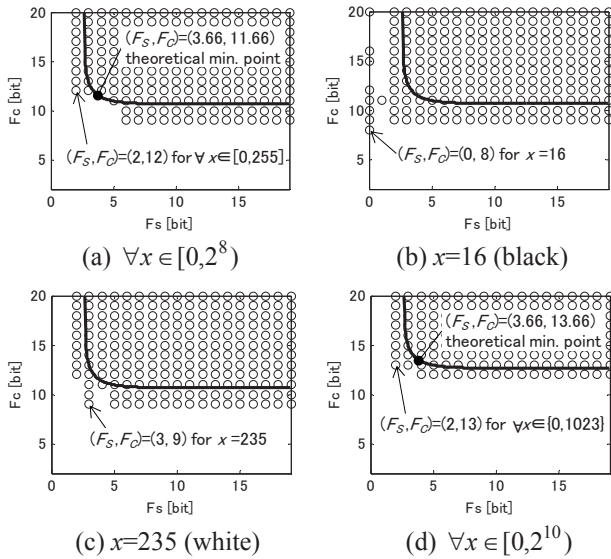


Fig.4 The word length for the DC Lossless DWT.

Table I The optimum word length for the DC lossless DWT. The cost function $J=F_C+F_S$ is minimized.

input DC values		signals	coefficients
		F_S [bit]	F_C [bit]
8 bit system	theoretical	4	12
	any $x \in [0, 255]$	2	12
	$x=16$ (black)	0	8
	$x=235$ (white)	3	9
10 bit system	theoretical	4	14
	any $x \in [0, 1023]$	2	13
	$x=64$ (black)	0	8
	$x=940$ (white)	0	12

The circles in Fig.4 (b) and (c) illustrate the points for a black value ($x=16$) and a white value ($x=235$) in the 8 bit system [10]. These results are summarized in table I. It is indicated that the word length is shortened by restricting the input value to specific values. For example, the optimum word length is $(F_S, F_C)=(3, 9)$ for the black value and the white value in the 8 bit system. Comparing to the case of $(F_S, F_C)=(4, 12)$, the cost function J is reduced from 16 to 12 [bit]. When the input value is restricted to only the black value, $(F_S, F_C)=(0, 8)$ with $J=8$ [bit]. It is indicated that the word length cost J can be halved for a practical case.

4. CONCLUSIONS

Introducing a new model which shifts the coefficient error to the signal error and utilizing the nullification of the accumulated errors, we theoretically derived the condition on the word length of signals and coefficients for the DC lossless DWT. It is indicated that the minimum cost function of the word length is halved by restricting the input DC value to a specific value in a practical case.

5. REFERENCES

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