

A New Class of Lifting Wavelet Transform for Guaranteeing Losslessness of Specific Signals

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Session: SPTM-L1.4 - Wavelets Analysis

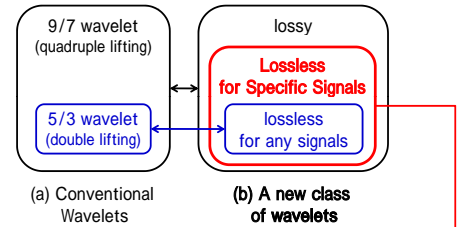
Presentation Type: Lecture

Session Location: Tiberius 1

Session Time: Tuesday, April 1, 10:30 - 12:30

Presentation Time: Tuesday, April 1, 11:30 - 11:50

Purpose of the research (1/2)

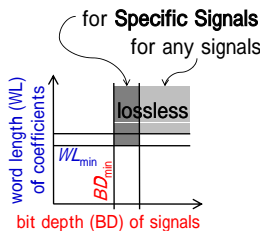


method	tap	lifting	input	property
conventional	5/3 9/7	double quadruple	any any	lossless lossy
new class	9/7	quadruple	any DC signals	lossy Specific lossless

Purpose of the research (2/2)

What to do?

- Determine the minimum
 - 1) bit depth (BD) of signals,
 - 2) word length (WL) of coefficients.



Rounding of a value

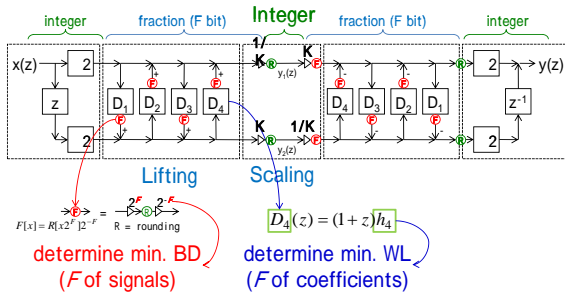
Original: $x = (-1)^S (b_{I-1} 2^{I-1} \dots + b_0 2^0 + b_{-1} 2^{-1} \dots + b_{-F} 2^{-F} + b_{-F-1} 2^{-F-1} + \dots)$
 Rounded: $x^* = (-1)^S (b_{I-1} 2^{I-1} \dots + b_0 2^0 + b_{-1} 2^{-1} \dots + b_{-F} 2^{-F})$
 S: sign I: integer F: fraction: F bit

$F[x] = R[x 2^F] 2^{-F}$

$R_{Ex1}[x] = \lfloor x + 2^{-1} \rfloor = \text{floor}(x + 2^{-1})$
 $R_{Ex2}[x] = \lceil x - 2^{-1} \rceil = \text{ceil}(x - 2^{-1})$
 R = rounding

- Determine the minimum
 - 1) bit depth (BD) -> F of signals
 - 2) word length (WL) -> F of coefficients

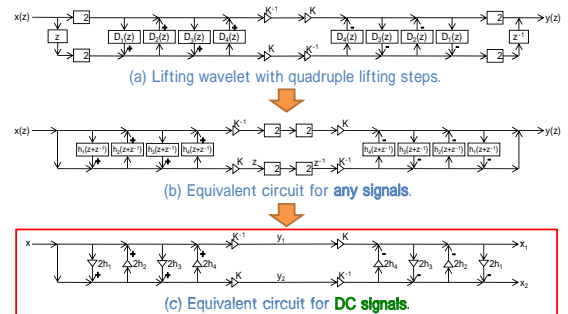
Lifting-Scaling / Integer Type LS(I) type



determine min. BD (F of signals)
 determine min. WL (F of coefficients)

A New Class = Lossless for DC signals
 DC signal = Example of specific signals

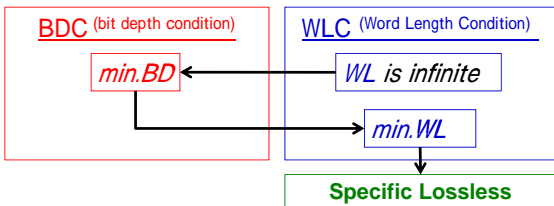
Equivalent Circuit for DC Signals



Use this circuit to determine min. BD and min. WL.

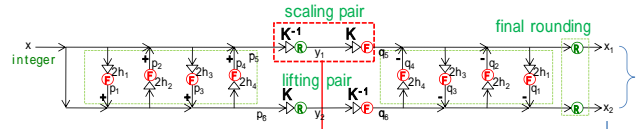
$D_1(z) \ D_2(z) = \begin{bmatrix} h_1 & h_2 \\ h_3 & h_4 \end{bmatrix} \begin{bmatrix} 1+z^{-1} & 0 \\ 0 & 1+z^{-1} \end{bmatrix}$
 $D_3(z) \ D_4(z) = \begin{bmatrix} h_1 & h_2 \\ h_3 & h_4 \end{bmatrix} \begin{bmatrix} 1+z^{-1} & 0 \\ 0 & 1+z^{-1} \end{bmatrix}$
 $K = +1.230174104914001$
 $K > 1$

Min. **BD** (bit depth) and **WL** (word length) for Specific Lossless



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BDC (bit depth condition) under infinite WL (1/2)



- 3. low scaling pair ($q_5 - p_5 = 0$)
- 4. high scaling pair ($q_6 - p_6 = 0$)
- 5. lifting pair ($q_i - p_i = 0, i=1,2,3,4$)

- 1. odd pass ($x_1 - x = 0$)
- 2. even pass ($x_2 - x = 0$)

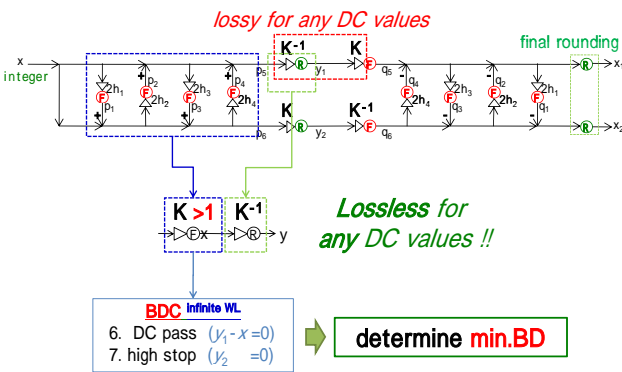


**can Not be Lossless
for any DC values !**

$R[K^{-1}x]$ is surjective &
 $F[Kx]$ is injective
--> $F[KR[K^{-1}x]]$ is not bijective

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BDC (bit depth condition) under infinite WL (2/2)



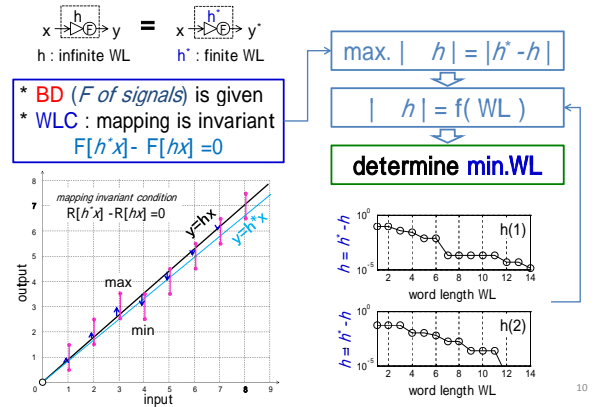
lossy for any DC values

**Lossless for
any DC values !!**

- 6. DC pass ($y_1 - x = 0$)
 - 7. high stop ($y_2 = 0$)
- determine min.BD**

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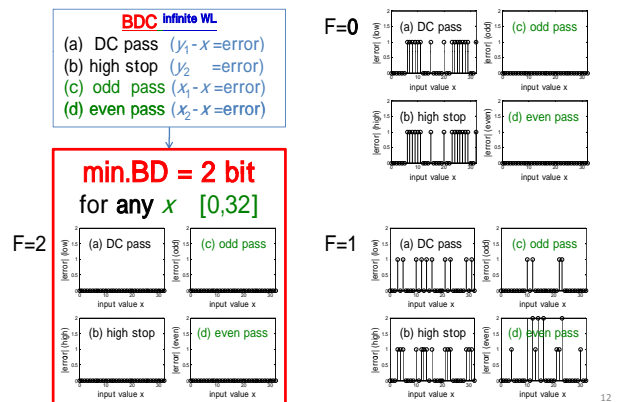
WLC (Word Length Condition) under min.BD



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Simulation Results

min.BD for Specific Lossless under infinite WL



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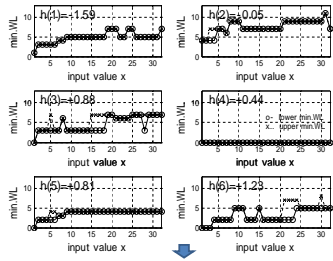
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min.WL for Specific Lossless under min.BD (F=2)

DC lossless for any value of $x \in [0,32]$

min. WL which satisfies all the conditions.

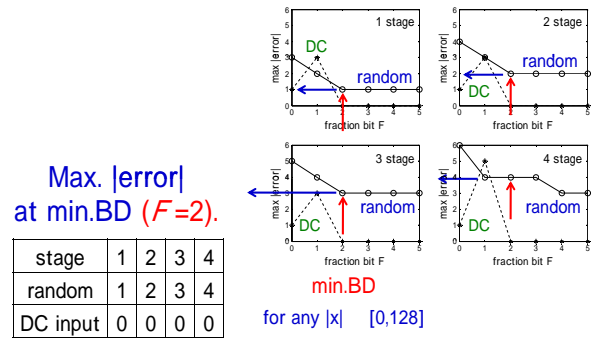
- (a) DC pass
- (b) high stop
- (c) odd pass
- (d) even pass



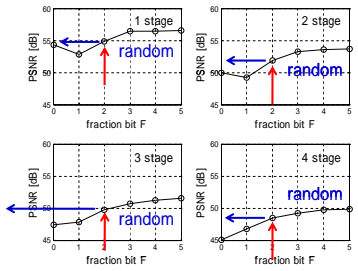
min.WL for any $x \in [0,32]$

	h(1)	h(2)	h(3)	h(4)	K^{-1}	K
random	7	12	14	0	4	13

Reconstruction error for random input (1/2)



Reconstruction error for random input (2/2)

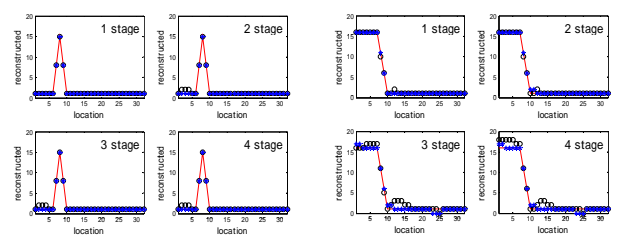


stage	1	2	3	4
random	54.87	51.86	49.75	48.47
DC input				

Negligible at PSNR < 48[dB]

Reconstruction error for impulse and step input

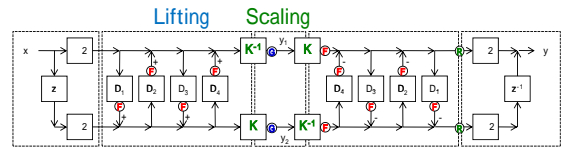
- input
- o with DC lossless
- * w/o DC lossless



tranERR²m

Variations of the Specific Lossless Wavelet

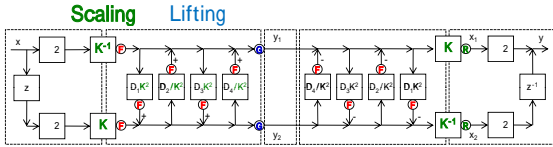
LS type conventional



(a) Lifting -Scaling (LS) type

	register	band	name
$F > 0, G = 0$	Fraction	Integer	LS(I) type
$F = G > 0$	Fraction	Fraction	LS(F) type

SL type Newly introduced



(b) Scaling-Lifting (SL) type

	register	band	name
$F > 0, G = 0$	Fraction	Integer	SL(I) type
$F = G > 0$	Fraction	Fraction	SL(F) type

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results of variations

method	input	min. BD		min. WL							
		F	G	forward		back		lifting			
				K^{-1}	K	K	K^{-1}	h_1	h_2	h_3	h_4
LS(F)	random	3		4	10	10	4	0			
SL(F)		1		4	9	10	4				

PROPOSAL

method	input	min. BD		min. WL							
		F	G	forward		back		lifting			
				K^{-1}	K	K	K^{-1}	h_1	h_2	h_3	h_4
LS(F)	DC signals	2		4	0	13	0	7	12	14	2
SL(F)		1		4	10	9	4	0			
LS(I)		2	0	4	0	13	0	7	12	14	0
SL(I)		3	0	4	10	10	4	11	14	7	0

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Conclusions

Proposed a New Class of Wavelet

- Specific Lossless *lossless for DC signals

Determined

- minimum BD for the Specific Lossless

- minimum WL under the min.BD

Standard requirement

- guarantee no error for white balance

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