IEEE International Symposium on Circuits and Systems (ISCAS) 2009, Taiwan, May 26, 2009

Three Dimensional Integer Rotation Transform and Improvement of its Compatibility

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***Abstract*— In this paper, a new structure of three dimensional integer rotation transform is proposed. The number of rounding operations is reduced from nine to four by introducing multi input lifting branches. Furthermore, the best combination of order of input signals is determined to avoid singular points of multiplier coefficients in the lifting branches. As a result, variance of errors due to rounding of signals into integers is reduced by 42.9 [%] at maximum and its compatibility to the conventional non integer rotation transform is improved.**

I. INTRODUCTION

Orthogonal transforms have a wide variety of applications such as the discrete cosine transform (DCT) in digital signal compression, the rotation transform in computer graphics and the principal component analysis (PCA) in statistical analysis. The PCA has been applied to color components to compress data size of an image signal [1,2]. It is required to reconstruct the image signal in its original form by its inverse procedure. However, the reconstructed signal is distorted when signals are rounded into integers inside the transform circuit in LSI.

This distortion can be avoided by introducing the lifting structure [3]. The original integer input signal is reconstructed without any loss. A two dimensional (2D) integer rotation transform (IRT) is composed of three lifting branches [4]. Since an orthogonal transform can be factorized into several 2D IRTs, it is also possible to construct a multi dimensional integer transform [5,6].

However, compatibility to the conventional non-integer transform is another problem to be discussed. When a signal is encoded by an integer (or non-integer) forward transform, and decoded by a non-integer (or integer) backward transform, the reconstructed signal contains rounding errors in general.

In this paper, we reduce the number of rounding operations of a three dimensional (3D) IRT by introducing multi-input lifting branches. In this case, some multiplier coefficients have singular points depending on its rotation angles. At the singular point, a coefficient value becomes infinity, so as the rounding error. To avoid this problem, we consider all the combinations of order of input signals, and determine the best combination. As a result, variance of the rounding error is reduced and the compatibility is improved.

II. EXISTING 3D INTEGER ROTATION TRANSFORM

A. Conventional Non-integer Rotation Transform

Relation between input values ***X*** and output values ***Y*** of a Givens-Jacobi type 2D rotation transform (RT) is defined by

$$\mathbf{Y} = \mathbf{R}_{2D}(\theta) \cdot \mathbf{X} \tag{1}$$

for

$$\mathbf{R}_{2D}(\theta) = \begin{bmatrix} \cos\theta & -\sin\theta \\ \sin\theta & \cos\theta \end{bmatrix}, \ \mathbf{X} = \begin{bmatrix} x_1 & x_2 \end{bmatrix}^T, \ \mathbf{Y} = \begin{bmatrix} y_1 & y_2 \end{bmatrix}^T \tag{2}$$

and θ is a rotation angle. Cascading the 2D RTs, the 3D RT with rotation angles $\theta_1, \theta_2, \theta_3$ is constructed as illustrated in figure 1. The transform is performed by

$$\mathbf{Y} = \mathbf{R}_{3D} \cdot \mathbf{X} \tag{3}$$

where

$$\mathbf{R}_{3D} = \begin{bmatrix} c_3 & 0 & -s_3 \\ 0 & 1 & 0 \\ s_3 & 0 & c_3 \end{bmatrix} \begin{bmatrix} 1 & 0 & 0 \\ 0 & c_2 & -s_2 \\ 0 & s_2 & c_2 \end{bmatrix} \begin{bmatrix} c_1 & -s_1 & 0 \\ s_1 & c_1 & 0 \\ 0 & 0 & 1 \end{bmatrix} \tag{4}$$

$$c_i = \cos\theta_i, \quad s_i = \sin\theta_i, \quad i \in \{1,2,3\}$$

for

$$\mathbf{X} = \begin{bmatrix} x_1 & x_2 & x_3 \end{bmatrix}^T, \ \mathbf{Y} = \begin{bmatrix} y_1 & y_2 & y_3 \end{bmatrix}^T. \tag{5}$$

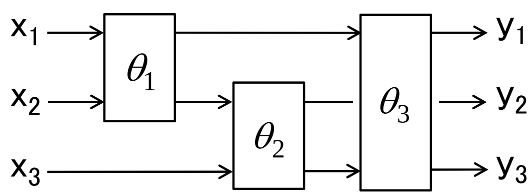

Figure 1. A three dimensional (3D) rotation transform (RT) with rotation angles $\theta_1, \theta_2, \theta_3$. The angles are determined by the PCA.

B. *Existing Integer Rotation Transform*

Figure 2 illustrates a 3D IRT. Each of the 2D RT in figure 1 is replaced by the 2D IRT composed of single-input lifting branches with a multiplier followed by a rounding operation denoted as "R". The multiplier coefficients $f_{ij},\ i, j \in \{1,2,3\}$ are determined so that they satisfy the equation:

$$\begin{bmatrix} 1 & 0 \\ f_{i3} & 1 \end{bmatrix}\begin{bmatrix} 1 & f_{i2} \\ 0 & 1 \end{bmatrix}\begin{bmatrix} 1 & 0 \\ f_{i1} & 1 \end{bmatrix} = \mathbf{R}_{2D}(\theta_i), \qquad i \in \{1,2,3\} \tag{6}$$

for each of the angles $\theta_1, \theta_2, \theta_3$. As a result, the coefficients are given by

$$\begin{bmatrix} f_{i1} & f_{i2} & f_{i3} \end{bmatrix} = \begin{bmatrix} (1-c_i)s_i^{-1} & -s_i & (1-c_i)s_i^{-1} \end{bmatrix}. \tag{7}$$

It should be noticed that magnitude of f_{i1} and f_{i3} become close to infinity when $\sin\theta_i$ approaches to zero. This is the singular point (SP) of the coefficients to be discussed.

All the difference (compatibility) between the IRT and the RT is due to the rounding operations. The existing 3D IRT in figure 2 contains nine. Error by the rounding is magnified by the coefficients in equation (7). Therefore not only the number of the rounding operations should be reduced, but also the SP should be carefully avoided, to increase the compatibility.

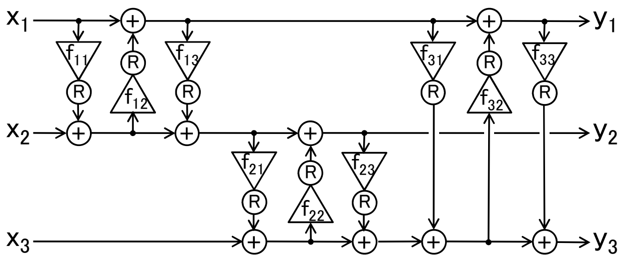

Figure 2. The existing 3D IRT. It includes nine rounding operations.

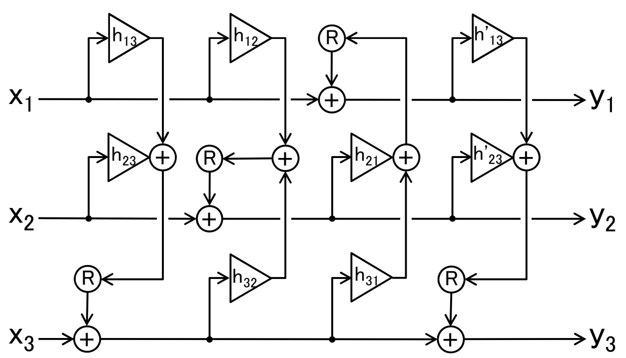

Figure 3. The proposed 3D IRT. The number of rounding is reduced from nine to four. Compatibility to the 3D RT is improved.

III. Proposed 3D Integer Rotation Transform

A. *Proposed Integer Rotation Transform*

Figure 3 illustrates the proposed 3D IRT. We introduce multi-input lifting branches. For example, the first lifting branch has two inputs x_1 and x_2. The number of rounding operations is reduced from nine to four. Total amount of the rounding error is expected to be reduced by the proposed IRT.

B. *Determination of the Coefficients*

The coefficients in figure 3 are determined so that they satisfy the equation:

$$\mathbf{H'}_3\mathbf{H}_1\mathbf{H}_2\mathbf{H}_3 = \mathbf{R}_{3D} \tag{8}$$

where

$$\mathbf{H'}_3 = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ h'_{13} & h'_{23} & 1 \end{bmatrix}, \quad \mathbf{H}_1 = \begin{bmatrix} 1 & h_{21} & h_{31} \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} \\ \mathbf{H}_2 = \begin{bmatrix} 1 & 0 & 0 \\ h_{12} & 1 & h_{32} \\ 0 & 0 & 1 \end{bmatrix}, \quad \mathbf{H}_3 = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ h_{13} & h_{23} & 1 \end{bmatrix}. \tag{9}$$

According to equation (4), $\mathbf{R}_{3D}$ is given by

$$\mathbf{R}_{3D} = \begin{bmatrix} c_3c_1 - s_3s_2s_1 & -c_3s_1 - s_3s_2c_1 & -s_3c_2 \\ c_2s_1 & c_2c_1 & -s_2 \\ s_3c_1 + c_3s_2s_1 & -s_3s_1 + c_3s_2c_1 & c_3c_2 \end{bmatrix} \tag{10}$$

Substituting equation (9) and (10) into (8), the coefficients are determined by

$$\begin{bmatrix} h_{32} \\ h_{23} \end{bmatrix} = \begin{bmatrix} -s_2 \\ (1-c_2c_1)/s_2 \end{bmatrix} \tag{11}$$

$$\begin{bmatrix} h_{21} \\ h_{31} \end{bmatrix} = \begin{bmatrix} 1+h_{32}h_{23} & h_{23} \\ h_{32} & 1 \end{bmatrix}^{-1} \begin{bmatrix} -c_3s_1 - s_3s_2c_1 \\ -s_3c_2 \end{bmatrix} \tag{12}$$

$$\begin{bmatrix} h_{12} \\ h_{13} \end{bmatrix} = \begin{bmatrix} h_{21} & h_{32}h_{21} + h_{31} \\ 1 & h_{32} \end{bmatrix}^{-1} \begin{bmatrix} c_3c_1 - s_3s_2s_1 - 1 \\ c_2s_1 \end{bmatrix} \tag{13}$$

$$\begin{bmatrix} h'_{13} \\ h'_{23} \end{bmatrix} = \begin{bmatrix} (\mathbf{F})_{11} & (\mathbf{F})_{21} \\ (\mathbf{F})_{12} & (\mathbf{F})_{22} \end{bmatrix}^{-1} \begin{bmatrix} s_3c_1 + c_3s_2s_1 - h_{13} \\ -s_3s_1 + c_3s_2c_1 - h_{23} \end{bmatrix} \tag{14}$$

where

$$\mathbf{F} = \mathbf{H}_1\mathbf{H}_2\mathbf{H}_3 \tag{15}$$

and $(\mathbf{F})_{ij}$ is a component of $\mathbf{F}$ at i-th column and j-th row:

$$\begin{bmatrix} (\mathbf{F})_{11} & (\mathbf{F})_{21} \\ (\mathbf{F})_{12} & (\mathbf{F})_{22} \end{bmatrix} = \\ \begin{bmatrix} 1 + h_{21}h_{12} + (h_{21}h_{32} + h_{31})h_{13} & h_{21} + (h_{21}h_{32} + h_{31})h_{23} \\ h_{32}h_{13} + h_{12} & 1 + h_{32}h_{23} \end{bmatrix} \tag{16}$$

C. Singular Point of the Coefficients

In the proposed 3D IRT, the coefficients have various singular points (SP). For example, according to equation (11), h_{23} has SP at $\theta_2 = \{0, \pm\pi, \pm 2\pi, \cdots\}$ where s_2 is zero. The coefficients [h_{21} h_{31}] do not have SP since

$$\begin{vmatrix} 1+h_{32}h_{23} & h_{23} \\ h_{32} & 1 \end{vmatrix} = 1\,. \qquad (17)$$

The coefficients [h_{12} h_{13}] have SP where

$$\begin{vmatrix} h_{21} & h_{32}h_{21}+h_{31} \\ 1 & h_{32} \end{vmatrix} = h_{31} = -s_3c_1 - c_3s_2s_1 \qquad (18)$$

becomes zero. Therefore, they have SP where

$$\cos\theta_1 \cos\theta_3 = 0 \quad or \quad \tan\theta_3 + \sin\theta_2 \tan\theta_1 = 0 \qquad (19)$$

holds. Similarly, [h'_{13} h'_{23}] have SP where

$$\begin{vmatrix} (\mathbf{F})_{11} & (\mathbf{F})_{21} \\ (\mathbf{F})_{12} & (\mathbf{F})_{22} \end{vmatrix} = 0 \qquad (20)$$

holds.

At these SPs, magnitude of the coefficients become close to infinity. Therefore, the rounding error is also amplified to close to infinity. This is the SP problem to be considered when the rotation angles in IRT are varied. This problem has not been well discussed for DCT since it has fixed coefficients [5]. However, in case of PCA where the angles changes depending on input signal's statistics, the IRT should be carefully constructed so that the SP problem can be avoided.

D. Combinations of Order of Input Signals

To avoid the SP problem, we consider all the combinations of order of input signals. Introducing the permutation matrix $\mathbf{P}_i$ where i indicates one of all the ${}_3P_3$= 6 combinations, we replace $\mathbf{R}_{3D}$ in equations (3) and (8) by

$$\mathbf{R}'_{3D} = \mathbf{P}_i^{-1}\mathbf{R}_{3D}\mathbf{P}_i\,, \qquad i \in \{1,2,3,4,5,6\} \qquad (21)$$

where

$$\mathbf{P}_1 = \begin{bmatrix} 1&0&0\\0&1&0\\0&0&1 \end{bmatrix},\quad \mathbf{P}_2 = \begin{bmatrix} 0&1&0\\1&0&0\\0&0&1 \end{bmatrix},\quad \mathbf{P}_3 = \begin{bmatrix} 1&0&0\\0&0&1\\0&1&0 \end{bmatrix},$$
$$\mathbf{P}_4 = \begin{bmatrix} 0&1&0\\0&0&1\\1&0&0 \end{bmatrix},\quad \mathbf{P}_5 = \begin{bmatrix} 0&0&1\\1&0&0\\0&1&0 \end{bmatrix},\quad \mathbf{P}_6 = \begin{bmatrix} 0&0&1\\0&1&0\\1&0&0 \end{bmatrix}, \qquad (22)$$

and we select the best combination that has the least error.

IV. Simulation Results

A. Determination of the Rotation Angles by the PCA

As an example, we determine the angles by the PCA. The input signals $\mathbf{X}$=[x_1, x_2, x_3]^T are set to color components {R, G, B} of an image signal. Table I summarizes the rotation angles $\theta_1, \theta_2, \theta_3$ for the input image "Lenna" at all the combinations in equation (22).

B. Evaluation of the Rounding Error

Variance of difference between output of the forward RT and that of the forward IRT is evaluated as the rounding error. Results are summarized in table II for "Lenna" at the combinations in table I. It is observed that the best combination of the existing IRT is (x_1,x_2,x_3)=(G,B,R) where variance of the rounding error is 0.416. In case of the proposed IRT, the best combination is (x_1,x_2,x_3)=(B,R,G) and variance is 0.289. The rounding error is reduced to 69.5 [%] (by 42.9 [%]) for the image "Lenna". Similarly, other images are examined. Results are summarized in table III. It is confirmed that the rounding error is reduced to 78.9, 85.1, 63.2 and 57.1 [%] for the image "Balloon", "Sail boat", "Milk drop" and "Pepper", respectively.

C. Theoretical Estimation of the Rounding Error

Denoting the error by the rounding operation in figure 2 as R_i, output of the forward existing IRT is calculated by

$$\begin{bmatrix} x'_1 \\ x'_2 \end{bmatrix} = \begin{bmatrix} 0 \\ R_3 \end{bmatrix} + \begin{bmatrix} 1 & 0 \\ f_{13} & 1 \end{bmatrix}\left(\begin{bmatrix} R_2 \\ 0 \end{bmatrix} + \begin{bmatrix} 1 & f_{12} \\ 0 & 1 \end{bmatrix}\left(\begin{bmatrix} 0 \\ R_1 \end{bmatrix} + \begin{bmatrix} 1 & 0 \\ f_{11} & 1 \end{bmatrix}\begin{bmatrix} x_1 \\ x_2 \end{bmatrix}\right)\right)$$
$$\begin{bmatrix} y_2 \\ x'_3 \end{bmatrix} = \begin{bmatrix} 0 \\ R_6 \end{bmatrix} + \begin{bmatrix} 1 & 0 \\ f_{23} & 1 \end{bmatrix}\left(\begin{bmatrix} R_5 \\ 0 \end{bmatrix} + \begin{bmatrix} 1 & f_{22} \\ 0 & 1 \end{bmatrix}\left(\begin{bmatrix} 0 \\ R_4 \end{bmatrix} + \begin{bmatrix} 1 & 0 \\ f_{21} & 1 \end{bmatrix}\begin{bmatrix} x'_2 \\ x_3 \end{bmatrix}\right)\right)$$
$$\begin{bmatrix} y_1 \\ y_3 \end{bmatrix} = \begin{bmatrix} 0 \\ R_9 \end{bmatrix} + \begin{bmatrix} 1 & 0 \\ f_{33} & 1 \end{bmatrix}\left(\begin{bmatrix} R_8 \\ 0 \end{bmatrix} + \begin{bmatrix} 1 & f_{32} \\ 0 & 1 \end{bmatrix}\left(\begin{bmatrix} 0 \\ R_7 \end{bmatrix} + \begin{bmatrix} 1 & 0 \\ f_{31} & 1 \end{bmatrix}\begin{bmatrix} x'_1 \\ x'_3 \end{bmatrix}\right)\right). \qquad (23)$$

On the other hand, the output of the proposed IRT in figure 3 is calculated by

$$\begin{bmatrix} y_1 \\ y_2 \\ y_3 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ R_4 \end{bmatrix} + \mathbf{H}'_3\left(\begin{bmatrix} R_3 \\ 0 \\ 0 \end{bmatrix} + \mathbf{H}_1\left(\begin{bmatrix} 0 \\ R_2 \\ 0 \end{bmatrix} + \mathbf{H}_2\left(\begin{bmatrix} 0 \\ 0 \\ R_1 \end{bmatrix} + \mathbf{H}_3\begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix}\right)\right)\right). \qquad (24)$$

Defining the total rounding error by

$$\mathbf{E} = [e_1 \quad e_2 \quad e_3]^T = [y_1 \quad y_2 \quad y_3]^T - [x_1 \quad x_2 \quad x_3]^T\,, \qquad (25)$$

it is calculated by

$$\mathbf{E} = \mathbf{H}_e[R_1 \quad R_2 \quad R_3 \quad R_4]^T \qquad (26)$$

where

$$\mathbf{H}_e = [\mathbf{H}'_3\mathbf{H}_1\mathbf{H}_2\mathbf{I}_L \quad \mathbf{H}'_3\mathbf{H}_1\mathbf{I}_M \quad \mathbf{H}'_3\mathbf{I}_U \quad \mathbf{I}_L]^T,$$
$$\mathbf{I}_U = \begin{bmatrix} 1\\0\\0 \end{bmatrix},\quad \mathbf{I}_M = \begin{bmatrix} 0\\1\\0 \end{bmatrix},\quad \mathbf{I}_L = \begin{bmatrix} 0\\0\\1 \end{bmatrix} \qquad (27)$$

in case of the proposed IRT.

Assuming that all the errors R_i are independent and identically uniformly distributed in the range of $[-2^{-1}, 2^{-1})$, total amount of the rounding error can be theoretically estimated by

$$\sigma_e^2 = \frac{1}{12} trace\left(\mathbf{H}_e{}^T \mathbf{H}_e\right). \qquad (28)$$

Results of this theoretical estimation are plotted in figure 4 as solid line and broken line for the existing IRT and the proposed IRT, respectively. The experimental results in table III are also plotted as circle and cross for the existing IRT and the proposed IRT, respectively. It is theoretically and experimentally confirmed that the proposed method reduces the rounding error by 42.9 [%] at maximum.

TABLE I. ROTATION ANGLES DETERMINED BY PCA FOR "LENNA".

$x_1-x_2-x_3$	R-B-G	B-R-G	R-G-B	G-R-B	B-G-R	G-B-R
theta 1	-32.91	-131.36	-48.59	156.69	-153.76	-135.27
theta 2	-43.59	15.90	-23.17	-46.21	-41.63	-19.30
theta 3	157.78	-134.20	-133.73	-28.52	-148.24	-49.89

TABLE II. VARIANCE OF THE ROUNDING ERROR AT ALL THE COMBINATIONS FOR THE IMAGE "LENNA".

$x_1-x_2-x_3$	R-B-G	B-R-G	R-G-B	G-R-B	B-G-R	G-B-R
existing 3D-IRT	0.972	0.546	0.417	0.906	1.105	**0.416**
proposed 3D-IRT	1.144	**0.289**	0.293	1.302	1.174	0.417

TABLE III. VARIANCE OF ROUNDING ERROR AT THE BEST COMBINATION FOR SEVERAL IMAGES.

$x_1-x_2-x_3$	Lenna	Balloon	Sailboat	milk drop	Pepper
existing 3D-IRT	0.416 (G-B-R)	0.616 (B-G-R)	0.464 (R-G-B)	0.348 (R-G-B)	0.289 (G-R-B)
proposed 3D-IRT	0.289 (B-R-G)	0.486 (G-R-B)	0.395 (G-B-R)	0.220 (B-R-G)	0.165 (B-R-G)

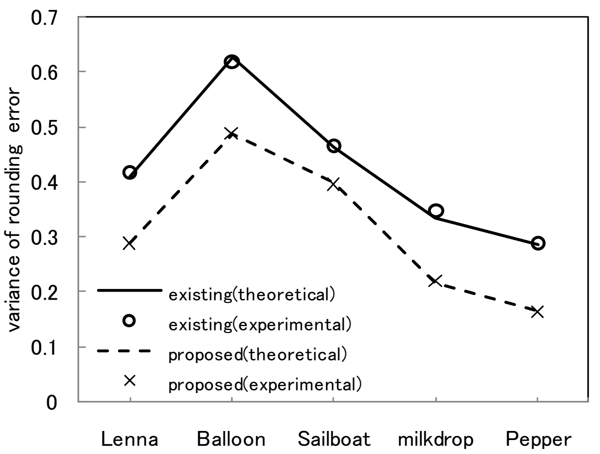

Figure 4. Variance of the rounding error at the best combination.

V. CONCLUSIONS

In this paper, we proposed a new structure of three dimensional integer rotation transform. The number of rounding operations is reduced from nine to four by introducing multi-input lifting branches. We pointed out the singular point problem of multiplier coefficients in the lifting branches of the 3D IRT. To avoid this problem, we determined the best combination of order of input signals so that the 3D IRT has the least amount of the rounding error. It is confirmed that the rounding error is reduced to 69.5, 78.9, 85.1, 63.2 and 57.1 [%] for the image "Lenna", "Balloon", "Sail boat", "Milk drop" and "Pepper", respectively. As a result, variance of the rounding errors is reduced by 42.9 [%] at maximum and compatibility of the 3D IRT to the conventional non-integer 3D RT is improved.

ACKNOWLEDGMENT

This research was partially supported by the Ministry of Education, Science, Sports and Culture, Grant-in-Aid for Scientific Research (C), 20560351, 2008.

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